The Role of Government in 
Child Care Provision

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Abstract
In this model, we analyze household preferences for parental and external child care as well as the effects of subsidized public child care and a parental child care benefit on welfare, fertility and investments in quality of children within a model with endogenous fertility and labor supply of a secondary earner. We find differences in the provision of quantity and quality of public child care. Our model predicts that both increased subsidies for public child care and the parental child care benefit have negative effects on fertility and the secondary earner's labor supply. Public child care and the parental child care benefit lead to a crowding-out of private external child care and are equally effective with respect to welfare for households choosing an interior solution of parental and external child care. We show that low income households choosing a corner solution of staying at home opt for a parental child care benefit if they have a small preference for children and their quality while for high income households differences in the quality for external child care are crucial.

JEL: H31, H53, J13, J22
Keywords: fertility, quality of children, public child care, quality of child care, secondary earner's labor supply, parental child care benefit

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1 Introduction

All European countries are facing dilemmas and tensions related to the complex interaction between changes in the labor market, fertility trends, and children’s opportunities. Social policy is therefore confronted with distinct policy objectives and instruments. In confronting the issue of low fertility levels and mothers labor market participation rates, policy makers have considered and tried a variety of options. One policy option that has received considerable attention is making high quality child care available and affordable (Attanasio et al., 2008).

In this paper, we present a theoretical model which analyzes preferences for parental and external child care and the effects of public policies on welfare, fertility and investments in quality of children within a model with endogenous fertility and labor supply of a secondary earner. We compare the households’ preferences for quantity and quality of publicly provided child care as well as for a price subsidy for publicly provided child care and a parental child care benefit. Publicly provided child care in this setting can either be provided by municipalities, churches or non-profit organizations while the carriers of privately provided external child care include for-profit institutions, child care by nannies, au-pairs or babysitters. Parents can opt for publicly provided child care or receive a parental child care benefit. This reflects the German system of parental child care benefits, the so called “Betreuungsgeld”, that are only obtainable for parents who either care for their children themselves or demand only privately provided external child care.¹ Parental child care benefits are also established in Finland, Norway, and Sweden but we focus on the German system. We analyze how effectively the policy instruments are able to achieve a compatibility of family and work and model the parents’ decision whether to demand publicly provided child care or receive a parental child care benefit.

The increased demand for child care accompanying the rise of two-earner couples has attracted the attention of policy makers and researchers alike. While the theoretical literature is relatively scarce, there exist various empirical studies on child care and maternal employment (see Blau and Currie (2006) for a survey).

¹ The “Betreuungsgeld” was introduced at the beginning of 2013 and is a monthly child care supplement for parents whose children aged three and under are not in a state-subsidized nursery. At the same time a spot in just such a nursery has become a legal entitlement for all children (Federal Ministry for Family Affairs, Senior Citizens, Women and Youth, 2013)
Borck and Wrohlich (2011) analyze preferences for a public, private or mixed provision of child care both theoretically and empirically. Similar to our theoretical model, they model child care as a publicly provided private good. Their model builds on the redistributive aspect of public provision of private goods (see Besley and Coate, 1991; Epple and Romano, 1996a). They conclude that richer households should prefer market provision to purely public or mixed provision while preferences over public versus mixed provision depend on the redistributive effect of public provision. We extend their approach by additionally considering parental child care and adding differences in the qualities of the child care options. Furthermore, we also extend their analysis by considering price subsidies for public child care and a parental child care benefit.

The results of empirical studies on the effect of public child care on fertility and female labor supply are ambiguous. Havnes and Mogstad (2011) evaluate the effect of child care on labor supply using a 1975 reform. They find that a large-scale expansion of subsidized child care in Norway had little impact on maternal employment.\(^2\) Instead, they found that the new subsidized child care mostly crowds out informal care arrangements implying a significant net cost of the child care arrangement. Previous research mostly came to the conclusion that affordable and readily available child care is a driving force both of cross-country differences in maternal employment and of its rapid growth over the last decades (see e.g. Jaumotte, 2003; Del Boca, 2002; Aaberge et al. 2005; Attanasio et al., 2008). Blau and Currie (2006) point out two fundamental problems of these studies: child care access and prices are endogenous to the work decision of mothers and the availability and cost of informal care is unobserved. Havnes and Mogstad (2011) conclude that the discrepancy between the estimates in the previous literature is likely to be explained by differences in biases owing to ignoring the substitution between subsidized and informal child care, misspecifications of functional forms for the employment and child care equations, and violations of the exclusion restriction.

There are also some empirical studies analyzing the effect of public child care on fertility. Rindfuss et al. (2010) examine a policy reform from the mid-1970s in Norway, which led to a substantial expansion of public child care. Their results suggest that moving from having no child care slots available for preschool children to having slots available for 60% of preschool children has a positive effect on fertility. Mörk et al. (2013) exploit the exogenous variation in parental fees caused by a Swedish child care reform in 2001 to identify the effect of child care

\(^2\) Subsidized child care in this case refers to public and private child care institutions, eligible for subsidies from the government because they satisfy federal quality requirements
costs on fertility. They find that the reduction in child care costs increased the number of first and higher order births, but only seemed to affect the timing of second births.

The second policy instrument we want to analyze is a parental child care benefit. Gathmann and Sass (2012) use a reform in East Germany that generated exogenous variation in child care prices to study the impact of child care costs on child care utilization, family labor supply and child well-being. In 2006, the government of Thuringia introduced a new family policy that provides generous subsidies to families who do not send their child to public child care. This family policy is very similar to the in 2013 introduced “Betreuungsgeld” in all German states which we formally analyze in our theoretical model. The specific structure of the subsidy in Thuringia is such that it declines linearly with the number of hours the eligible child attends public child care. As such, the subsidy is equivalent to an increase in the hourly price of public child care (fully compensated by an income subsidy). Gathmann and Sass (2012) find that parents respond to those changes in the price for public child care by reducing both public and informal daycare and increasing parental child care. Declines in public daycare attendance are especially dramatic for children from low-skilled, single parents and low income families. Their results suggest that the decline in female labor force participation is strongest for single parents and low income households while they find no effect on fertility in eligible households.

Our main findings are the following. We find that for households opting for public child care, an increase in the quantity of publicly provided child care has a negative effect on both fertility and secondary earner’s labor supply while the effects of an increase in its quality depend on the price effect and may be positive. Our results also suggest that both a price subsidy for publicly provided child care and the parental child care benefit has a negative effect on fertility and the secondary earner’s labor supply for all households.

Regarding welfare, we find that for households choosing an interior solution of parental and external child care both policy instruments, a parental child care benefit and publicly provided child care, are equally effective. The household’s decision which policy instrument to choose only depends on the relative benefit the household receives in total. This decision is independent of the household’s income. For households choosing a corner solution, on the other hand, the decision depends on a variety of parameters: their income, their preferences for children, child care and consumption as well as on the policy parameters. Low income households choosing a corner solution of staying at home and not consuming external child care opt for a parental child care benefit if they have a small preference for children and their
quality. For high income households, the decision depends on differences between the qualities of private and public external child care. If the relative benefit a household receives from the policy instruments is identical and the quality of private external child care is larger than the quality of publicly provided child care, a household in this corner solution also benefits more from the parental child care benefit than from publicly provided child care.

The paper is organized as follows. In section 2 we introduce the economic setup. Section 3 presents the comparative static results for households choosing an interior solution of parental and external child care as well as for households choosing a corner solution. In section 4, we calculate the welfare effects of the parents’ decision whether or not to demand publicly provided child care and section 5 concludes.

2 The model

We use a static model and allow for heterogeneity in the households’ preferences with respect to consumption, fertility, and the child’s quality. Family $i$’s decisions are assumed to be taken by the parents who derive utility from their own consumption, $c_i$, their number of children, $n_i$, and their child’s quality of life, $q_i$. The household's preferences are represented by the following utility function

$$U_i = (1 - \gamma_i) \ln c_i + \gamma_i \ln(n_i q_i)$$

with $\gamma_i \in \{0,1\}$ and $i \in \{1, \ldots, N\}$. The parameter $\gamma_i$ captures the family’s relative weight given to their children. Note that households care about both child quantity and child quality. The quality per child, $q_i$, can be understood as a good produced domestically by the parents who use as inputs time spent with the child and a child-specific consumption good, $z_i$, bought on the market. For simplicity, we assume that only the secondary earner of family $i$ spends time with the children. Time spent with a child can be divided into the secondary earner’s own time, $h_i$, the time the child spends at privately provided external child care, $s_i$, and the time the child spends at publicly provided external child care, $g$. We assume that publicly provided child care in this setting can either be provided by municipalities, churches or non-profit organizations while the carriers of privately provided child care include for-profit institutions, child care by nannies, au-pairs or babysitters. Parents can decide whether they
want to make demands on publicly provided child care or not and there is thus a possibility to opt out. If parents decide not to demand public child care, they are eligible for a parental child care benefit. In case parents opt for public child care, we assume that all children receive the same quantity of public child care in this economy. All households are allowed to supplement the publicly provided quantity and the quantity of private purchase may differ across households. It should be noted that the fact that the quantity of public child care is common to all children whose parents opt for public child care does not mean that all households receive the same quantity of public services, because the number of children may differ across households. The strictly concave domestic production function for a child’s quality is given by

\[ q_i = \theta(z_i)^{\eta_i}(1 + h_iq_h)^{\rho_i}(1 + \alpha_i g q_g + s_i q_s)^{(1-\rho_i)} \]  

(2)

with \( \theta > 0 \), \( \eta_i \in \{0,1\} \), and \( \rho_i \in \{0,1\} \). The parameter \( \rho_i \) illustrates the household’s preference for parental child care and \( (1 - \rho_i) \) shows the household’s preference for external child care. \( \eta_i \) depicts the household’s preference for child-specific consumption. This quality technology thus assumes diminishing returns to both child-specific consumption and parental child care time, but constant returns to scale in parental and external child care time. The parameter \( \alpha_i \) represents the parents’ decision regarding public child care. For \( \alpha_i = 0 \) the parents decide against while for \( \alpha_i = 1 \) they opt for publicly provided child care. The terms \( (1 + h_i q_h) \) and \( (1 + \alpha_i g q_g + s_i q_s) \) guarantee that the child’s quality remains positive even if parents either do not care for their child themselves or do not demand external child care. Note that we allow for corner solutions regarding parental and external child care time but not regarding child-specific consumption. All forms of child care time are assumed to have different qualities: \( q_h \) describes the quality of the secondary earner to care for her child while \( q_g \) and \( q_s \) depict the quality of the public and private daycare teachers, respectively.

The secondary earner allocates her time to working which yields income at the rate \( w_i \) and to leisure time. We assume that child rearing is the only domestic time requiring parental time so that she spends her leisure time completely with the children. Through the endogeneity of \( n_i \), the secondary earner’s labor supply is also endogenous. If she has \( n_i \) children her parental time equals \( h_i n_i \). The rest of secondary earner \( i \)’s total time is working time and given by \( L_i = 1 - h_i n_i \), her average lifetime gross income therefore equals \( w_i L_i \). Secondary earners carrying a larger wage rate, \( w_i \), thus have higher opportunity costs of both having and raising children. The primary earner allocates all her time to working and her average lifetime gross income equals \( w_i L_i \).
salary is $Y_i$. The market prices for privately provided child care, $s_i$, and publicly provided child care, $g$, are denoted by $\pi_s$ and $\pi_g$, respectively. We assume that the government subsidizes the price for publicly provided child care by $\beta$. The price on the market for the child-specific consumption good, $z_i$, is denoted by $B$. Parents can choose whether they want to make demands on publicly provided child care or not. If they choose to opt out, they receive a parental child care benefit, $\sigma$, per child. The decision is denoted by the parameter $\alpha_i$. For $\alpha_i = 0$ the family decides to take a parental child care benefit while for $\alpha_i = 1$ they opt for publicly provided child care. The family’s budget constraint is thus given by:

\[(Y_i + w_i L_i)(1 - t) = c_i + \pi_s s_i n_i + \alpha_i \pi_g (1 - \beta) gn_i + B z_i n_i - (1 - \alpha_i) \sigma n_i \quad (3)\]

Parents choose consumption, $c_i$, the number of children, $n_i$, the secondary earner’s time spent with a child, $h_i$, the amount of bought-in child care, $s_i$, as well as child-specific consumption, $z_i$, so as to maximize their utility, $U_i(c_i, n_i, q_i)$, by taking account of the child’s quality production and their budget constraint.

The household decision problem is given by:

\[
\max_{c_i, n_i, q_i, z_i, h_i, s_i} U_i(c_i, n_i, q_i(z_i, h_i, s_i, g)) \quad s. t. \quad (3)
\]

(4)

Following Kimura and Yasui (2009), we assume that the technology for converting expenditures into quantity of service is the same as that in the private sector. The government’s budget constraint is

\[
t[\bar{Y} + \bar{w}(1 - \bar{h}\bar{n})] = \bar{\alpha} \beta \bar{n} g \pi_g + (1 - \bar{\alpha}) \sigma \bar{n} \quad (5)
\]

where $\bar{\alpha}$ denotes the average decision for publicly provided child care or the parental child care benefit, $\bar{n}$ the average number of children, $\bar{h}$ the average parental child care time, and $\bar{w}$ and $\bar{Y}$ the average secondary earner’s wage rate and primary earner’s income in the population, respectively. For simplification we denote $\bar{m}$ as the average labor income of a family in the population, $[\bar{Y} + \bar{w}(1 - \bar{h}\bar{n})]$. We assume that the government and the private market both set their prices for child care according to their marginal costs. The variable costs for providing one unit of external child care, $k(q_x)^x$, incur for employing additional daycare teachers and thus depend on the quality of the child care center, with $x \in \{s, g\}$. Hence, we assume that child care centers can increase their quality by recruiting additional daycare teachers. The prices for external child care are therefore given by
\[ \pi_g = k(q_g)^\kappa \]  \hspace{1cm} (6)  
\[ \pi_s = k(q_s)^\kappa \]  \hspace{1cm} (7)

with \( k > 0 \) and \( \kappa > 0 \). The government’s budget constraint in (5) can thus be written as
\[ t\bar{m} = \bar{\alpha}\beta g\bar{n}k(q_g)^\kappa + (1 - \bar{\alpha})\sigma\bar{n}. \]

Hence, in our analysis, the government has four instruments: the quantity and quality of publicly provided child care, the price subsidy as well as the parental child care benefit. In the following we will analyze the impact of the four instruments on fertility, the secondary earner’s labor supply, the child’s quality, and on welfare.

The solution to the household decision problem in (4) can either be interior, or at a corner where the household chooses either not to work and not to demand private external child care or to work fulltime and to demand the maximum amount of private external child care. For households choosing an interior solution of parental and external child care time, the first-order conditions imply\(^3\)

\[ n_i^* = \frac{\gamma \eta_i (Y_i + w_i)(1 - t)}{P_{n,i}} \]  \hspace{1cm} (8)  
\[ h_i^* = \frac{\rho_i P_{n,i}q_h - \eta_i w_i (1 - t)}{\eta_i w_i (1 - t)q_h} \]  \hspace{1cm} (9)  
\[ s_i^* = \frac{(1 - \rho_i) P_{n,i} q_s - \eta_i \pi_s (1 + \alpha_i g q_g)}{\eta_i \pi_s q_s} \]  \hspace{1cm} (10)  
\[ z_i^* = \frac{P_{n,i}}{B} \]  \hspace{1cm} (11)  
\[ q_i^* = \frac{\theta (1 - \rho_i) P_{n,i} q_s (P_{n,i})^{\eta_i} \left( \frac{\rho_i \pi_s q_h}{B} \right) \left( \frac{\rho_i \pi_s q_h}{(1 - \rho_i) w_i (1 - t) q_s} \right)^{\rho_i}}{\eta_i \pi_s q_s} \]  \hspace{1cm} (12)

where \( P_{n,i} \) is the relative price for child care:
\[ P_{n,i} \equiv \frac{w_i (1 - t)}{q_h} + \frac{\pi_s}{q_s} + \frac{\alpha_i g (\pi_s q_g - \pi_g (1 - \beta) q_s)}{q_s} + (1 - \alpha_i)\sigma \]  \hspace{1cm} (13)

As we assume private and public external child care to be substitutes, the household’s benefit from public child care depends on the relative price difference. In the following, we assume

\(^3\)See Appendix B for the derivations.
that the government sets the price subsidy for publicly provided child care, $\beta$, such that $[\pi_s q_g \pi_g (1 - \beta) q_s] \text{ holds. This implies that the government always sets the subsidy such that } 1 > \beta > -\left(\frac{\pi_s}{\pi_g}\right)^{1/(1-\kappa)} \text{ holds. The government therefore has to consider differences in the quality of private and public child care when setting the price for public child care. Hence, both publicly provided child care and a parental child care benefit increase the relative price for child care, $P_{n,i}$, in (13). The reason for this increase in the relative price is that public child care effectively increases the price of private external child care while the parental child care benefit effectively increase the price of public child care for parents.}

The second-order conditions for a maximum in (8) – (11) are satisfied. Due to increasing opportunity costs of having children, parental child care decreases in the secondary earner’s wage rate. The demand for external child care, on the other hand, increases in the secondary earner’s wage rate due to the costs associated with buying external child care. Depending on the size of $\rho_i$ and $\eta_i$, parents might decide for a corner solution of caring solely alone for their children or not at all. For households with a relatively high preference for parental child care, there is a threshold wage rate, $\hat{\omega}_{it}^h$, below which secondary earners choose a corner solution of not buying external child care, $s_i^* = 0$. For households with a relatively small preference for parental child care, there is a threshold wage rate, $\hat{\omega}_{iu}^h$, above which secondary earners choose not to stay at home with the children such that $h_i^* = 0$. We additionally assume that the maximum demand for external child care time is one. Parents can neither buy more than one unit of external child care time per child nor spend more time at home with the child. There is thus also a threshold wage rate, $\hat{\omega}_{it}^s$, above which households cannot consume more external child care. At maximum they can consume one unit of privately provided external child care. If the households additionally demand public child care, the maximum amount of private child care decreases to $(1 - g)$. We furthermore assume that the demand for parental child care time is at maximum either $\frac{1}{n_i}$ or one depending on whether the family has more than one child or not. The secondary earner’s labor supply cannot be negative, such that $h_i n_i \leq 1$ must hold true. There is thus a threshold wage rate, $\hat{\omega}_{it}^s$, below which the secondary earner cannot spend more time with the child at home. Even though parents do not consume external child care below the threshold wage rate of $\hat{\omega}_{it}^s$, they can still increase their number of children in our model. With an increasing fertility, the average maximum time the secondary earner can
spend with each child decreases. Parental child care might therefore increase in the secondary earner’s wage rate for households with a wage rate below $w_{iL}$. The thresholds wage rates are

$$s^*_i = 0: \quad \tilde{w}_{iL}^s = \frac{q_h \left[ \eta_i \pi_s \left(1 + \alpha_i gq_g\right) - (1 - \rho_i)P_{n|i=0} \right]}{(1 - \rho_i)(1 - t)} \quad (14)$$

$$s^*_i = 1 - \alpha_i g: \quad \tilde{w}_{iL}^s = \frac{q_h \left[ \eta_i \pi_s \left(1 + q_s + \alpha_i g(q_g - q_s)\right) - (1 - \rho_i)P_{n|i=0} \right]}{(1 - \rho_i)(1 - t)} \quad (15)$$

$$h^*_i = 0: \quad \tilde{w}_{iL}^h = \frac{\rho_i P_{n|i=0} q_h}{(\eta_i - \rho_i)(1 - t)} \quad (16)$$

$$h^*_i = 1: \quad \tilde{w}_{iL}^h = \frac{\rho_i P_{n|i=0} q_h}{\eta_i (1 + q_h) - \rho_i}(1 - t) \quad (17)$$

with $P_{n|i=0} \equiv \frac{\pi_s}{q_s} + \frac{\alpha_i g \pi_s q_g - \pi_g (1 - \beta) q_s}{q_s} + (1 - \alpha_i) \sigma$. The households’ threshold levels thus depend critically on the decision whether to demand publicly provided child care or not as well as on the size of the family policy instruments and the household’s preferences. Changes in the family policy instruments shift the threshold levels to the right or left and therefore influence the household’s decision for labor supply.

Using the results described above, we can depict the relationship between the demand for parental and privately provided external child care and the secondary earner’s wage rate as follows:

![Figure 1](image-url): The relationship between the demand for parental and privately provided external child care and the secondary earner’s wage rate (for $\alpha_i = 0$)

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4 See Appendix A for a detailed function of parental and external childcare time.
These relationships are consistent with the well-known evidence: high income households choose relatively low parental child care times with relatively high investments in external child care. Note that we do not make an assumption on total child care time. Parental and external child care time can therefore be in sum smaller or larger than one.

In case the household has more than one child, the threshold $\tilde{\sigma}_{LU}$ shifts to the right and the maximum amount of parental child care decreases to $\frac{1}{n_i}$. For households choosing public child care, the maximum amount of external child care they can demand decreases to $(1 - g)$ and the threshold $\tilde{\sigma}_{LU}$ shifts to the left.

3 Comparative statics: The effects of changes in the family policies

In the following, we analyze the comparative statics results for changes in the policy parameters for households choosing an interior solution as well as for households choosing a corner solution of parental and external child care.

3.1 Interior solution of parental and external child care

For households choosing an interior solution of parental and external child care, we find that fertility in (8) decreases in the secondary earner’s wage rate as long as

\[
\left\{ \frac{\gamma_i(1 - \ell)}{q_h} > \frac{\pi_s}{q_s} - \frac{\alpha_i g \gamma_i - \gamma_i q_h}{q_s} (1 - \alpha_i \sigma) \right\}.^5
\]

That is if the relative income of the primary earner is larger than the relative costs for external child care. The lowest and highest possible fertility rates in this case are given by $\lim_{w_i \to \infty} n_i^* = \gamma_i \eta_i q_h$ and $\lim_{w_i \to 0} n_i^* = \frac{\gamma_i \eta_i (1 - \ell)}{p_{n_i(w_i = 0)}}$. In our model, families with a positive preference for children, $\gamma_i$, a positive parental child care quality, $q_h$, and a positive preference for child-specific-consumption, $\eta_i$, will thus always have children. Assuming that the primary earner’s income is sufficiently high, we can depict the relationship between fertility and the secondary earner’s wage rate as follows:

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$^5$ See Appendix C for the remaining derivations in this section.
Figure 2: The relationship between fertility and the secondary earner’s wage rate

In case of a single household, that is $Y_i = 0$, fertility would increase in the households’ income.

An increase in the secondary earner’s parental quality, $q_h$, on the other hand, always has a positive effect on fertility. Regarding the quality of external child care, we find that an increase in the quality of privately provided child care in (18) has a positive effect on fertility for $0 \leq \kappa \leq 1$:

$$\frac{\partial n_i}{\partial q_s} = \frac{y_i \eta_i Y_i (1 - t)}{(P_{n,i} | w = 0)^2(q_s)^2} \frac{(1 + \alpha_i g q_g)}{(P_{n,i} | w = 0)^2(q_s)^2}$$

An increase in the quality of publicly provided child care as shown in (19), on the other hand, has an ambiguous effect on the number of family $i$’s children. For a family choosing not to demand publicly provided child care, that is $\alpha_i = 0$, an increase in the quality of public child care always has a negative effect on fertility due to the tax increase. For a family choosing $\alpha_i = 1$, however, an increase in the quality of publicly provided child care might also have a positive effect on fertility depending on its impact on the price, that is on $\kappa$. For $0 \leq \kappa \leq 1$, the effect of an increase in the quality of publicly provided child care on fertility is also negative for households opting for public child care.

$$\frac{\partial n_i^*}{\partial q_g} = -\Lambda_i g \left\{ \alpha_i m (1 - t) \left[ \frac{\pi_s}{q_s} - \frac{\kappa \pi_g (1 - \beta)}{q_g} \right] + \frac{\kappa \alpha \beta \pi_g}{q_g} \left[ P_{n,i} - \frac{w_i (1 - t)}{q_h} \right] \right\}$$

$$\frac{\partial n_i^*}{\partial g} = -\Lambda_i q_g \left\{ \alpha_i m (1 - t) \left[ \frac{\pi_s}{q_s} - \frac{\pi_g (1 - \beta)}{q_g} \right] + \frac{\alpha \beta \pi_g}{q_g} \left[ P_{n,i} - \frac{w_i (1 - t)}{q_h} \right] \right\}$$
\[
\frac{\partial n_i^*}{\partial \beta} = -\Lambda_i g \pi_g \left\{ \alpha_i \bar{m}(1-t) + \bar{a} \bar{n} \left[ P_{n,i} - \frac{w_i(1-t)}{q_h} \right] \right\} \\
\frac{\partial n_i^*}{\partial \sigma} = -\Lambda_i \left\{ (1-\alpha_i) \bar{m}(1-t) + (1-\bar{a}) \bar{n} \left[ P_{n,i} - \frac{w_i(1-t)}{q_h} \right] \right\}
\]

with \( \Lambda_i \equiv \frac{\gamma_i \eta_i^{(Y_i+w_i)}}{\bar{m}(p_{n,i})^2} > 0 \).

Regarding an increase in the quantity of publicly provided child care in (20), the effect is negative for all households. For \( \kappa = 1 \), the effects of an increase in quality and in quantity on fertility only differ with respect to the initial level of \( g \) and \( q_g \). The effects of an increase in the price subsidy, \( \beta \), in (21), and of an increase in parental child care benefit, \( \sigma \), in (22), on fertility are also negative for all households. The reasons for these negative effects of the policy instruments on fertility are the following. Assuming \( \pi_s q_g > \pi_g (1-\beta) q_s \), an increase in publicly provided child care effectively increases the price for private external child care and thus increases the parents relative price for child care. As shown in (13), the parental child care benefit also increases the household’s relative price for child care and thus decreases the parent’s demand for children.

Regarding parental time in (9), we find that an increase in the secondary earner’s wage rate leads to a decrease in parental child care time for all families. Assuming \( \eta_i > \rho_i \), an increase in parental child care quality, on the other hand, has a positive effect on parental child care. Concerning the quality of external child care, we find that an increase in the quality of privately provided child care leads to a decrease in parental child care time for \( 0 \leq \kappa \leq 1 \) and thus increases the secondary earner’s labor supply:

\[
\frac{\partial h_i^*}{\partial q_s} = -\frac{\rho_i(1-\kappa)\pi_s(1+\alpha_i g q_g)}{\eta_i w_i(q_s)^2(1-t)}
\]

An increase in the quality of publicly provided child care as shown in (24), however, has a positive effect on parental child care for all families choosing \( \alpha_i = 0 \). For families choosing \( \alpha_i = 1 \), the impact is ambiguous and depends on price effect, that is on \( \kappa \). For \( 0 \leq \kappa \leq 1 \), the effect of an increase in the quality of publicly provided child care has a negative effect on the secondary earner’s labor supply for all households.

\[
\frac{\partial h_i^*}{\partial q_g} = \Gamma_i g \left\{ \alpha_i \bar{m}(1-t) \left[ \frac{\pi_s}{q_s} - \frac{\kappa \pi_g (1-\beta)}{q_g} \right] + \frac{\kappa \bar{a} \bar{n} \beta \pi_g}{q_g} \left[ P_{n,i} - \frac{w_i(1-t)}{q_h} \right] \right\}
\]
\[
\frac{\partial h^*_i}{\partial g} = \Gamma_i q_g \left\{ \alpha_i \tilde{m} (1-t) \left[ \frac{\pi_s}{q_s} - \frac{\pi_g (1-\beta)}{q_g} \right] + \bar{\alpha} \tilde{\beta} \pi_g \left[ P_{n,i} - \frac{w_i (1-t)}{q_h} \right] \right\} 
\]

(25)

\[
\frac{\partial h^*_i}{\partial \beta} = \Gamma_i \left\{ \alpha_i \tilde{m} (1-t) + \bar{\alpha} \tilde{\beta} \left[ P_{n,i} - \frac{w_i (1-t)}{q_h} \right] \right\} 
\]

(26)

\[
\frac{\partial h^*_i}{\partial \sigma} = \Gamma_i \left\{ (1-\alpha_i) \tilde{m} (1-t) + (1-\bar{\alpha}) \tilde{\beta} \left[ P_{n,i} - \frac{w_i (1-t)}{q_h} \right] \right\} 
\]

(27)

with \( \Gamma_i \equiv \frac{\rho_i}{\eta_i \bar{m} w_i (1-t)^2} > 0 \).

An increase in the quantity of publicly provided child care in (25) has a positive effect on parental child care for all households and thus decreases the secondary earner’s labor supply. As before, for \( \kappa = 1 \), the effects of an increase in quality and in quantity only differ with respect to the initial level of \( g \) and \( q_g \). The same effect as for an increase in the quantity of publicly provided child care applies to an increase in the price subsidy, \( \beta \), in (26) and an increase in the parental child care benefit, \( \sigma \), in (27). Our model thus predicts that an increase in both the subsidy for publicly provided child care and the parental child care benefit has a negative effect on the secondary earner’s labor supply for all households. Due to the negative income effect caused by the increase in the income tax, the secondary earner’s opportunity costs of staying at home with the children and not working decrease and the secondary earner therefore increases her parental time. The size of the effects therefore decreases in the secondary earners wage rate. (Compare to Figure 1)

Summarizing our results with respect to the reconciliation of family and work for households choosing an interior solution of parental and external child care, we find that only an increase in the quality of privately provided external child care has a positive effect on both fertility and the secondary earner’s labor supply for all households if \( 0 \leq \kappa \leq 1 \) holds. Even though an increase in the quality of private external child care also leads to an increase in the price for child care, parents increase their demand for children and at the same time increase their labor supply. For households demanding public child care, that is \( \alpha_i = 1 \), an increase in both the quantity of publicly provided child care and the price subsidy for publicly provided child care has a negative effect on fertility and secondary earner’s labor supply. The effect of an increase in the quality of publicly provided child care, however, depends on \( \kappa \) and is positive for \( \kappa > 1 \). Concerning the parental child care benefit, our model predicts a negative effect on both fertility and the secondary earner’s labor supply for all households choosing an interior solution of parental and external child care.
**Proposition 1:** An increase in the quantity of publicly provided child care, the subsidy for public child care as well as in the parental child care benefit has a negative effect on both fertility and the secondary earner’s labor supply for all households. Increasing the quality of publicly provided child care, however, may increase both fertility and the secondary earner’s labor supply for households opting for public child care depending on the price effect.

Regarding the demand for privately provided external child care in (4.10), we find that an increase in the secondary earner’s wage rate has a positive effect while an increase in parental child care quality has a negative impact. An increase in the quality of privately provided child care has an ambiguous effect on the demand for external child care and depends on the price effect of an increase in the quality, that is on $\kappa$, as well as on the household’s preferences for parental child care and child-specific consumption:

$$\frac{\partial s_i^*}{\partial q_s} = -\frac{1}{\eta_i\pi_s q_s q_h} \left[ (1 - \rho_i) \left[ \kappa P_{n,i} + (1 - \kappa) \frac{\pi_s}{q_s} (1 + \alpha_i g q_g) \right] - \eta_i \pi_s \frac{1}{q_s} (1 + \alpha_i g q_g) \right]$$

(28)

The same holds true for an increase in both the quality and quantity of publicly provided external child care as shown in (29) and (30) for families choosing $\alpha_i = 1$. In both cases, the effects are negative if $(\eta_i + \rho_i) > 1$ holds. Parents then use publicly provided child care as a substitution for private external child care. For families choosing $\alpha_i = 0$, the effects are in both cases negative due to the increase in the income tax. Therefore, an increase in both the quantity and the quality of publicly provided child care leads to a crowding-out of privately provided child care for households opting for the parental child care benefit and likely also for households opting for public child care.

$$\frac{\partial s_i^*}{\partial q_g} = \frac{(1 - \rho_i) g [\alpha_i \bar{m} q_h (\pi_s q_g - \kappa g (1 - \beta) q_s) - \kappa \bar{a} \bar{b} \bar{w}_i \pi_g q_g] - \eta_i \alpha_i g \bar{m} \pi_s q_h q_g}{\eta_i \bar{m} \pi_s q_h q_s q_g}$$

(29)

$$\frac{\partial s_i^*}{\partial g} = \frac{(1 - \rho_i) [\alpha_i \bar{m} q_h (\pi_s q_g - \pi_g (1 - \beta) q_s) - \bar{a} \bar{b} \bar{w}_i \pi_g q_s] - \eta_i \alpha_i \bar{m} \pi_s q_h q_g}{\eta_i \bar{m} \pi_s q_h q_s}$$

(30)

$$\frac{\partial s_i^*}{\partial \beta} = \frac{(1 - \rho_i) g \pi_g (\alpha_i \bar{m} q_h - \bar{a} \bar{w}_i)}{\eta_i \bar{m} \pi_s q_h}$$

(31)

$$\frac{\partial s_i^*}{\partial \sigma} = \frac{(1 - \rho_i) [(1 - \alpha_i) \bar{m} q_h - (1 - \bar{a}) \bar{w}_i]}{\eta_i \bar{m} \pi_s q_h}$$

(32)
An increase in the subsidy for publicly provided external child care, $\beta$, in (31) has also an ambiguous effect on the demand for privately provided child care for families having decided to demand public child care, that is $\alpha_i = 1$. The income effect is positive for low income earners while it is negative for high income earners due to the associated increase in the income tax. The effect is thus more likely to be negative the larger the secondary earner’s wage rate. The impact of an increase in $\beta$ also depends on the average income, the average demand for public child care, and the average number of children. For families opting for the parental child care benefit, that is $\alpha_i = 0$, the income effect is negative and households thus demand less privately provided external child care. In case of an increase in the parental child care benefit in (32), however, the effect is ambiguous for households choosing $\alpha_i = 0$ while the effect is negative for households choosing $\alpha_i = 1$. The effect for households opting for the parental child care benefit is more likely to be positive for households whose secondary earner carries a low wage rate as for them the income effect is again positive. For households who decided to demand publicly provided child care, we observe a decrease in the demand for privately provided child care due to the associated increase in the income tax. This negative effect therefore increases in the household’s secondary earner’s wage rate.

Summarizing our results for households choosing to demand publicly provided child care, that is $\alpha_i = 1$, we find that there is a crowding-out effect with respect to all policy instruments for secondary earners with a high wage rate due to the negative income effect. As shown before, only an increase in the quality of publicly provided child care may have a positive effect on the secondary earner’s labor supply while for all other instruments the effect is negative. Parents thus do not use the publicly provided child care to increase their labor supply but rather substitute private external child care time.

For households choosing $\alpha_i = 0$, we also observe a crowding-out effect of private external child care for all three instruments influencing publicly provided child care due to the negative income effect. In this case, only an increase in the parental child care benefit may lead to a stronger demand for private external child care but at the same time it also has a negative effect on the secondary earner’s labor supply. Both effects are stronger for low income households. Low income households opting for the parental child care benefit therefore use additional parental child care benefit payments to decrease their labor supply while they at the same time increase their demand for private external child care.
Proposition 2: An increase in both the quantity and quality of publicly provided child care has a negative effect on the household’s demand for private external child care. An increase in the subsidy for publicly provided child care (parental child care benefit) has a positive effect on the household’s demand for private external child care for low income households opting for public child care (the parental child care benefit).

Our model predicts that child-specific consumption in (11) increases in the secondary earner’s wage rate and decreases in parental child care quality. An increase in the quality of privately provided child care has a negative effect on child-specific consumption due to the increased price for external child care. For families opting for the parental child care benefit, that is \( \alpha_i = 0 \), an increase in both the quantity and quality of publicly provided child care has a negative effect. For households choosing \( \alpha_i = 1 \), however, the effect is in both cases ambiguous but more likely to be positive for low income households due to the redistribution via the income tax. The same applies for an increase in the price subsidy for publicly provided child care. An increase in the parental child care benefit, on the other hand, has a negative effect on child-specific consumption for families choosing \( \alpha_i = 1 \) while the effect is ambiguous for families choosing \( \alpha_i = 0 \) and also depends on the secondary earners wage rate.

Regarding the child’s quality in (12), we find that an increase in the secondary earner’s wage rate has a positive effect. An increase in parental child care quality, on the other hand, has a negative effect on the child’s quality. Concerning the quality of external child care in (33), we find that the impact of an increase depends crucially on the price effect, that is on \( \kappa \), as well as on the household’s preferences for parental and child-specific consumption. The effect of an increase in \( q_s \) is therefore ambiguous.

\[
\frac{\partial q_i^*}{\partial q_s} = \frac{\phi_i (1 - \kappa)}{q_s} \left\{ (1 - \rho_i)P_{n,i}q_s - (1 + \eta_i)\pi_s (1 + \alpha_i g q_g) \right\}
\]  

(33) with \( \phi_i \equiv \frac{\theta(1-\rho_i)}{\eta_i \pi_s} \left( \frac{P_{n,i}}{B} \right)^{\eta_i} \left( \frac{\rho_i \sigma_h q_h}{(1-\rho_i)w_i(1-\epsilon)q_s} \right)^{\rho_i} > 0 \).

An increase in the quality of public child care in (34), is more likely to have a positive impact on the child’s quality for families demanding public child care, that is \( \alpha_i = 1 \). The same applies for the effects of an increase in the quantity of public child care in (35) and an increase in the price subsidy, \( \beta \), in (36). All three effects are also more likely to be positive for secondary earners with a low wage rate due to the redistribution via the income tax and a
large parental child care quality. As an increase in the policy instruments leads to an increase in parental child care, the quality of this care is crucial for the child’s quality. The effects also depend on the average demand for public child care and the average fertility.

\[
\frac{\partial q_i^*}{\partial q_g} = \frac{\phi_i g}{\bar{m} q_g (1 - t)} \{\rho_i \kappa \bar{\alpha} \bar{\beta} \bar{\pi}_n q_s q_h - \kappa (1 - \beta) q_s \} - \kappa \bar{\alpha} \bar{\beta} \bar{\pi}_n q_g q_s (1 - t) + (1 + \eta_i) [\alpha_i \bar{m} q_h (\pi_s q_g - \pi_g (1 - \beta) q_s) - \bar{\alpha} \bar{\beta} \bar{\pi}_n q_g q_s] (1 - t) \]

(34)

\[
\frac{\partial q_i^*}{\partial g} = \frac{\phi_i}{\bar{m} q_h (1 - t)} \{\rho_i \alpha \bar{\alpha} \bar{\pi}_n q_n q_g q_s + (1 + \eta_i) [\alpha_i \bar{m} q_h (\pi_s q_g - \pi_g (1 - \beta) q_s) - \bar{\alpha} \bar{\beta} \bar{\pi}_n q_g q_s] (1 - t) \}

(35)

\[
\frac{\partial q_i^*}{\partial \beta} = \frac{\phi_i q_s}{\bar{m} q_h (1 - t)} \{\rho_i (1 - \alpha) \bar{\pi}_n q_n q_h + (1 + \eta_i) [(1 - \alpha_i) \bar{m} q_h - \bar{\alpha} \bar{w}_i] (1 - t) \}

(36)

\[
\frac{\partial q_i^*}{\partial \sigma} = \frac{\phi_i q_s}{\bar{m} q_h (1 - t)} \{\rho_i (1 - \alpha) \bar{\pi}_n q_n q_h + (1 + \eta_i) [(1 - \alpha_i) \bar{m} q_h - (1 - \bar{\alpha}) \bar{w}_i] (1 - t) \}

(37)

The effect of an increase in the parental child care benefit, \( \sigma \), in (37), on the other hand, is more likely to be positive for households opting for a parental child care benefit, that is \( \alpha_i = 0 \). The impact in this case is more likely to be positive for secondary earners carrying a low wage rate and having a large parental child care quality. As before, the reason for this is the redistribution via the tax system and the importance of the parental child care quality considering the increase in parental child care.

**Proposition 3:** For low income households with a large parental child care quality opting for public child care, an increase in both the quantity and quality of as well as in the subsidy for publicly provided child care is likely to have a positive effect on the child’s quality. The same applies to households opting for a parental child care benefit with respect to an increase in the parental child care benefit.

### 3.2 Corner solutions of parental and external child care

Next to the results for households choosing an interior solution of parental and external child care, we also analyze the effects with respect to the children’s quality for households choosing a corner solution. We take two different cases into consideration: first, secondary
earners who stay at home with their children and do not consume any private external child care; second, we also consider households who do not spend any time with their children and consume the maximum amount of external child care. In the first case, we have to differentiate between two groups of households: those with one child at maximum and those with more than one child. In households with \( n_i \leq 1 \) the secondary earner can at most spend one unit of time at home, that is \( h_i = 1 \), while in households with \( n_i > 1 \) the secondary earner can only spend at most \( \frac{1}{n_i} \) units of time per child at home. The crucial aspect is that in both cases the secondary earner’s labor supply is zero, \( L_i = 1 - h_i n_i = 0 \), for the period of time she stays at home with her children. In the second case, the maximum amount of privately provided external child care the household can consume depends on the household’s decision whether to demand public child care or not and is thus \( 1 - \alpha_i g \).

Choosing a corner solution for child care only affects the households’ quality choice for their children. We assume that the fertility choice depends on the secondary earner’s average lifetime wage rate and is therefore not affected by her decision for this specific period of time. The first-order conditions for the children’s quality in the two cases imply

\[
q_i^* \mid h_i = 1, s_i = 0 = \theta \left( 1 + \alpha_i g q_g \right) \left( \frac{P_{n,i}}{B} \right)^{\eta_i} \left( \frac{1 + q_h}{1 + \alpha_i g q_g} \right)^{\rho_i} \tag{38}
\]

\[
q_i^* \mid h_i = \frac{1}{n_i}, s_i = 0 = \theta \left( 1 + \alpha_i g q_g \right) \left( \frac{P_{n,i}}{B} \right)^{\eta_i} \left( \frac{Y_i \eta_i (Y_i + w_i) (1 - t) + P_{n,i} q_h}{Y_i \eta_i (1 + \alpha_i g q_g) (Y_i + w_i) (1 - t)} \right)^{\rho_i} \tag{39}
\]

\[
q_i^* \mid h_i = 0, s_i = 1 - \alpha_i g = \theta \left[ 1 + q_s - \alpha_i g (q_s - q_g) \right] \left( \frac{P_{n,i}}{B} \right)^{\eta_i} \left( \frac{1}{1 + q_s - \alpha_i g (q_s - q_g)} \right)^{\rho_i} \tag{40}
\]

where \( P_{n,i} \) is the relative price for child care as described in (13).

Regarding the child’s quality for the case that a household decides to stay full-time at home in (38), we find that, in contrast to the case of the interior solution, the effect of an increase in parental child care quality is ambiguous as the household cannot further increase parental child care.\(^6\) An increase in the quality of public child care in (41) has a negative effect for households opting for a parental child care benefit due to the increase in the income tax. The impact for households demanding public child care is likely to be positive for households choosing this corner solution as it depends on the secondary earner’s wage rate and

\(^6\) See Appendix D for the remaining derivations in this section.
households choosing this corner solution usually carry a relatively small wage rate. The same applies to an increase in the quantity of publicly provided child care in (42). The effects also critically depend on the secondary earner’s parental child care quality as in the case of households choosing an interior solution of parental and external child care.

\[
\frac{\partial q_i^*|_{h_i=1,s_i=0}}{\partial q_g} = \phi_i|_{h_i=1,s_i=0} \left\{ (1 - \rho_i) \alpha_i g \\
+ \frac{\eta_i g}{\bar{m}P_{n,i}q_h q_g} (1 + \alpha_i g q_g)[\alpha_i \bar{m}q_h (\pi_s q_g - \kappa \pi_g (1 - \beta) q_s) \right\} 
\]

\[
\frac{\partial q_i^*|_{h_i=1,s_i=0}}{\partial g} = \phi_i|_{h_i=1,s_i=0} \left\{ (1 - \rho_i) \alpha_i g \\
+ \frac{\eta_i}{\bar{m}P_{n,i}q_h q_g} (1 + \alpha_i g q_g)[\alpha_i \bar{m}q_h (\pi_s q_g - \pi_g (1 - \beta) q_s) \right\} 
\]

\[
\frac{\partial q_i^*|_{h_i=1,s_i=0}}{\partial \beta} = \eta_i \phi_i|_{h_i=1,s_i=0} \frac{g \pi_g}{\bar{m}P_{n,i}q_h} (1 + \alpha_i g q_g)(\alpha_i \bar{m}q_h - \bar{a} \bar{n} w_i) 
\]

\[
\frac{\partial q_i^*|_{h_i=1,s_i=0}}{\partial \sigma} = \eta_i \phi_i|_{h_i=1,s_i=0} \frac{g \pi_g}{\bar{m}P_{n,i}q_h} (1 + \alpha_i g q_g)(1 - \alpha_i) \bar{m}q_h - (1 - \alpha) \bar{a} \bar{n} w_i 
\]

with \(\phi_i|_{h_i=1,s_i=0} = \theta \left( \frac{P_{n,i}}{\beta} \right) \eta_i \left( \frac{1+q_h}{1+\alpha_i g q_g} \right)^{\rho_i} > 0\).

An increase in the subsidy for public child care, \(\beta\), in (43) is also likely to have a positive effect on the child’s quality for households opting for public child care, while we observe a negative effect for households choosing a parental child care benefit. Increasing the parental child care benefit in (44), on the other hand, has a negative effect for households choosing \(\alpha_i = 1\) while the effect is likely to be positive for households choosing \(\alpha_i = 0\). Both effects are again likely to be positive as they depend on the secondary earners wage rate. As households in this corner solution choose the maximum amount of parental child care time, the effects also critically depend on the parental child care quality.

The results are very similar for the case of a household with more than one child in (39) and can be found in Appendix D.

**Proposition 4:** For a household choosing a corner solution of staying at home and opting for public child care, an increase in both the quantity and quality of as well as in the subsidy for
publicly provided child care is likely to have a positive effect on the child’s quality. For a household opting for a parental child care benefit, an increase in the parental child care benefit is in this corner solution likely to have a positive effect on the child’s quality. In both cases the effects crucially depend on the secondary earner’s parental child care quality.

In case of secondary earners with a relatively high wage rate choosing a corner solution of consuming external child care and abstaining from parental child care in (40), we find that the child’s quality increases in the secondary earner’s wage rate. The impact of an increase in the quality of private external child care in (45) is positive for $\kappa \geq 1$ and otherwise depends on the household’s preferences for parental child care and child-specific consumption:

$$\frac{\partial d_i^*}{\partial q_s} \bigg|_{h_i=0,s_i=1-\alpha_i g} = \phi_i \left(1 - \rho_i\right) \left(1 - \alpha_i g\right)$$

$$- \eta_i (1 - \kappa) \pi_s \left[1 + q_s \left(1 - \alpha_i g + \alpha_i g q_g\right) \left(1 + \alpha_i g q_g\right)\right]$$

$$P_n(q_s)^2 \left(1 - \rho_i\right).$$

An increase in the quality of public child care in (46) has a negative effect for households opting for the parental child care benefit due to the increase in the tax rate. The effect for households demanding public child care is ambiguous and also depends on the price effect as well as on the households’ preferences for child-specific consumption and parental child care. The same applies to an increase in the quantity of public child care in (47). For households with a small preference for parental child care - this should be the case for households choosing this corner solution - opting for publicly provided child care, the effect with respect to the quantity is likely to be negative if $q_g < q_s$. In this case, an increase in the quantity of publicly provided child care decreases the maximum amount of private child care the households can consume and the total quality of external child care therefore decreases.
One main difference in the results for this corner solution compared to the one where households do not consume external child care is that increases in both the subsidy for public child care (see (48)) and the parental child care benefit (see (49)) are likely to have a negative effect on the child’s quality for all households. Secondary earners choosing this corner solution of abstaining from parental child care carry relatively high wage rates and thus do not benefit from the redistribution via the income tax. Hence, they do not benefit from increasing subsidies or parental child care benefits. Both results also depend on the secondary earner’s parental child care quality. The larger the secondary earner’s parental child care quality, the smaller is the negative effect on the child’s quality. This relationship can be explained by the effect of the policy instruments on fertility. The more children a family has, the more it benefits from redistribution via the number of children.
**Proposition 5**: For a household choosing a corner solution of consuming the maximum amount of private external child care and working full-time, an increase in both the subsidy for publicly provided child care and the parental child care benefit is likely to have a negative effect on the child’s quality. For a household opting for publicly provided child care, the effect of an increase in the quantity of public child care in this corner solution depends on the qualitative differences between public and private child care.

4 Welfare analysis

In the welfare analysis, we analyze the redistribution effects of the four policy instruments on households with different incomes and preferences. We assume that the benevolent government maximizes the household’s indirect utility function $V_i(g, q_g, \beta, \sigma)$ subject to the government’s budget constraint in (5). For households choosing an interior solution of parental and external child care time, we find the following indirect utility levels for the two options of choosing publicly provided child care or a parental child care benefit:

\[ V_{i|\alpha=0} = (1 - \gamma_i) \ln[(1 - \gamma_i)(Y_i + w_i)(1 - t)] \]

\[ + \gamma_i \ln \left[ \frac{\gamma_i \theta (1 - \rho_i)}{\pi_s} (Y_i + w_i)(1 - t) q_s \right] \]

\[ - t q_s \left( \frac{P_{n,i|\alpha=0}}{B} \right)^{\eta_i} \left( \frac{\rho_i \pi_s q_h}{(1 - \rho_i)(1 - t) q_s} \right)^{\rho_i} \]  

(50)

\[ V_{i|\alpha=1} = (1 - \gamma_i) \ln[(1 - \gamma_i)(Y_i + w_i)(1 - t)] \]

\[ + \gamma_i \ln \left[ \frac{\gamma_i \theta (1 - \rho_i)}{\pi_s} (Y_i + w_i)(1 - t) q_s \right] \]

\[ - t q_s \left( \frac{P_{n,i|\alpha=1}}{B} \right)^{\eta_i} \left( \frac{\rho_i \pi_s q_h}{(1 - \rho_i)(1 - t) q_s} \right)^{\rho_i} \]  

(51)

The policy parameters do not influence the parent’s own consumption but only their child-specific consumption choices. Households always adjust child care time such that there is no difference in the two scenarios in (50) and (51) with this respect. Due to the income effect, the households’ endogenous decision on $\alpha$ influences their decision concerning child-specific consumption. We thus find that for a household choosing an interior solution of parental and
external child care the only crucial factor is the size of the respective policy instrument and therefore the relative price for child care, $P_{n,i}$. Household $i$’s indirect utility is identical in both situations as long as the parental child care benefit equals the relative benefit from publicly provided child care:

$$V_{l|α_i=0} = V_{l|α_i=1} \iff \sigma = \frac{g[\pi_s q_g - \pi_g (1 - \beta) q_s]}{q_s}$$

(52)

For households choosing an interior solution of parental and external child care, both policy instruments, the parental child care benefit and publicly provided child care, are thus equally effective with respect to welfare and their decision which instrument to choose only depends on the relative benefit the household receives in total. Households choosing an interior solution of parental and external child care will thus opt for public child care as long as $\{g[\pi_s q_g - \pi_g (1 - \beta) q_s] > \sigma q_s\}$ holds independent of their income.

**Proposition 6:** For a household choosing an interior solution of parental and external child care time, publicly provided child care and the parental child care benefit are equally effective with respect to welfare. The household’s decision for a policy instrument in this case only depends on the relative benefit the household receives from the policy instrument in total.

For households choosing a corner solution, changes in the policy parameters do not only influence the parents’ child-specific consumption choices but also child care options and their own consumption possibilities in this period. In case of secondary earners choosing a corner solution of not consuming private external child care and not working, the household’s consumption possibilities in this period are decreased as the secondary earner’s labor supply is zero. Thus, the only source of income is the primary earner’s net income. Next to the parents’ preference for children and their quality, $γ_i$, parental consumption in this case also depends on their preference for child-specific consumption, $η_i$, and the relative price for child care, $P_{n,i}$. We find the following indirect utility levels for the first case with $h_i = 1$ and $s_i = 0$ for the two scenarios:
\[ V_{i|\alpha_i=0, h_i=1, s_i=0} = (1 - \gamma_i) \ln \left\{ Y_i(1 - t) - \frac{\gamma_i \eta_i(Y_i + w_i)(1 - t)}{P_{n_i|\alpha_i=0}} \left[ \frac{w_i(1 - t)}{q_h} + \frac{\pi_s}{q_s} \right] \right\} \\
+ \gamma_i \ln \left[ \frac{\gamma_i \eta_i \theta}{P_{n_i|\alpha_i=0}} (Y_i + w_i)(1 - t) \left( \frac{P_{n_i|\alpha_i=0}}{B} \right) \eta_i (1 + q_h)^{\rho_i} \right] \] (53)

\[ V_{i|\alpha_i=1, h_i=1, s_i=0} = (1)
- \gamma_i \ln \left\{ Y_i(1 - t) - \frac{\gamma_i \eta_i(Y_i + w_i)(1 - t)}{P_{n_i|\alpha_i=1}} \left[ \frac{w_i(1 - t)}{q_h} + \frac{\pi_s(1 + g q_g)}{q_s} \right] \right\} \\
+ \gamma_i \ln \left[ \frac{\gamma_i \eta_i \theta (1 + g q_g)}{P_{n_i|\alpha_i=1}} (Y_i + w_i)(1 - t) \left( \frac{P_{n_i|\alpha_i=1}}{B} \right) \eta_i (1 + q_h)^{\rho_i} \right] \] (54)

For households with \( n_i > 1 \) who are therefore choosing a corner solution with \( h_i = \frac{1}{n_i} \) and \( s_i = 0 \), we find the following indirect utility functions for the two scenarios:

\[ V_{i|\alpha_i=0, h_i=\frac{1}{n_i}, s_i=0} = (1 - \gamma_i) \ln \left\{ Y_i(1 - t) - \frac{\gamma_i \eta_i(Y_i + w_i)(1 - t)}{P_{n_i|\alpha_i=0}} \left[ \frac{w_i(1 - t)}{q_h} + \frac{\pi_s}{q_s} \right] \right\} \\
+ \gamma_i \ln \left[ \frac{\gamma_i \eta_i \theta}{P_{n_i|\alpha_i=0}} (Y_i + w_i)(1 - t) \left( \frac{P_{n_i|\alpha_i=0}}{B} \right) \eta_i (1 + q_h)^{\rho_i} \right] \] (55)

\[ V_{i|\alpha_i=1, h_i=\frac{1}{n_i}, s_i=0} = (1)
- \gamma_i \ln \left\{ Y_i(1 - t) - \frac{\gamma_i \eta_i(Y_i + w_i)(1 - t)}{P_{n_i|\alpha_i=1}} \left[ \frac{w_i(1 - t)}{q_h} + \frac{\pi_s(1 + g q_g)}{q_s} \right] \right\} \\
+ \gamma_i \ln \left[ \frac{\gamma_i \eta_i \theta (1 + g q_g)}{P_{n_i|\alpha_i=1}} (Y_i + w_i)(1 - t) \left( \frac{P_{n_i|\alpha_i=1}}{B} \right) \eta_i (1 + q_h)^{\rho_i} \right] \] (56)

Note that the consumption possibilities of the parents are identical for the two cases in (53) and (55) as well as in (54) and (56). The only difference is the child care time which depends
on the maximum amount of parental child care time available to the parents. The parents’
decision whether to opt for the parental child care benefit or public child care in these
scenarios not only depends on the relative benefit they receive but also on their preferences
and their income. Parental consumption in this corner solution is always larger in case the
household opts for the parental child care benefit as this policy is a direct payment which
increases parental income. Households with a low preference for children and their quality,
\( \gamma_i \), therefore benefit more from a parental child care benefit than from publicly provided child
care. For households with a relatively large preference for children and their quality, \( \gamma_i \), the
decision also depends on the household’s preference for parental child care.

**Proposition 7:** A household with a large preference for parental consumption choosing a
corner solution of not consuming private external child care and not working benefits more
from a parental child care benefit than from additional publicly provided child care.

In case parents choose a corner solution of abstaining from parental child care and consuming
the maximum amount of external child care, their consumption possibilities are also
influenced in this period due to the time restrictions.

\[
V_{i|\alpha_i=0, h_i=0, s_i=1} = (1 - \gamma_i) \ln \left( (Y_i + w_i)(1 - t) \right) - \gamma_i \eta_i \left( \frac{(Y_i + w_i)(1 - t)}{q_h} \left( \frac{\pi_s (1 + q_s)}{q_s} \right) + \frac{\pi_s (1 + q_s)}{q_s} \right)
\]

\[
+ \gamma_i \ln \left( \frac{\eta_i}{P_{n,i|\alpha_i=0}} \left( Y_i + w_i \right)(1) \left( \frac{P_{n,i|\alpha_i=0}}{B} \right) \eta_i \left( \frac{1}{1 + q_s} \right)^{\rho_i} \right)
\]  

\[
V_{i|\alpha_i=1, h_i=0, s_i=1-g} = (1 - \gamma_i) \ln \left( (Y_i + w_i)(1 - t) \right) - \gamma_i \eta_i \left( \frac{(Y_i + w_i)(1 - t)}{q_h} \left( \frac{\pi_s (1 + q_s + g(q_g - q_s))}{q_s} \right) \right)
\]

\[
+ \gamma_i \ln \left( \frac{\eta_i}{P_{n,i|\alpha_i=1}} \left( Y_i + w_i \right)(1 - t) \left( \frac{P_{n,i|\alpha_i=1}}{B} \right) \eta_i \left( \frac{1}{1 + q_s + g(q_g - q_s)} \right)^{\rho_i} \right)
\]  

\[
(57, 58)
\]
Whether parental consumption is increased or decreased depends on the parents’ preference for (child-specific) consumption. Parental consumption in (57) and (58) therefore also depends on the parents’ preference for children and child-specific consumption as well as on the relative price for child care.

Contrary to the circumstances before, the secondary earner in this case does work full-time and therefore her wage rate is relevant for parental consumption. Hence, the parents’ decision whether to opt for a parental child care benefit or publicly provided child care in this corner solution also not only depends on the relative benefit they receive but also on their preferences and their income. As opposed to the scenarios discussed before, parental consumption in this corner solution is not always larger in case the household opts for a parental child care benefit. In this case, the household’s decision for a policy option depends on the qualitative differences between private and public external child care. In all three corner solutions, welfare in the two scenarios is not equal if the relative benefits from the parental child care benefit and public child care are identical, that is if \( g(\pi_s q_g - \pi_g (1 - \beta) q_s) = \sigma q_s \) holds. This condition is not sufficient for households choosing a corner solution of parental and external child care. For households choosing a corner solution of abstaining from parental child care and consuming the maximum amount of external child care, the two scenarios are equal if the relative benefits from the parental child care benefit and public child care are identical and at the same time \( q_s = q_g \) holds. As long as \( q_s > q_g \), the households always benefits more from the parental child care benefit than from publicly provided child care.

**Proposition 8**: For a household choosing a corner solution of consuming the maximum amount of private external child care and working full-time, the decision whether to opt for a parental child care benefit or publicly provided child care depends on the qualitative differences between private and public external child care. If the relative benefit the household receives from the policy instruments in total is identical and \( q_s > q_g \) holds, the household prefers the parental child care benefit to publicly provided child care.

We can therefore conclude that while for households choosing an interior solution the decision whether to opt for the parental child care benefit or public child care only depends on the relative benefit the households receives from the policy option, the decision is much more
complex for households choosing a corner solution. In case of corner solutions, the household’s decision depends not only on the relative benefit but also on the household’s income and her preferences for children as well as for consumption and child care.

5 Conclusion

Summarizing our comparative statics results regarding the reconciliation of family and work for households choosing an interior solution of parental and external child care, we find that for households opting for public child care, an increase in the quantity of publicly provided child care has a negative effect on both fertility and secondary earner’s labor supply, while the effects of an increase in its quality depend on the price effect and may be positive. For households opting out of publicly provided child care and thus demanding a parental child care benefit, increases in both the quantity and the quality of public child care have negative effects on fertility and the secondary earner’s labor supply because of the increase in the income tax. Our results suggest that both a price subsidy for publicly provided child care and the parental child care benefit has a negative effect on fertility and the secondary earner’s labor supply for all households.

Our finding with respect to the parental child care benefit is similar to Apps and Rees (2004) and Gathmann and Sass (2012). Apps and Rees (2004) show that a parental child care benefit – or in their case a child benefit - may have a negative effect on fertility. Gathmann and Sass (2012) find that an increase in the parental child care benefit has a negative effect on female labor force participation and that the decline is strongest for single parents and low income households.

Our results suggest that only an increase in the quality of privately provided external child care may have a positive effect on both fertility and the secondary earner’s labor supply for all households. Even though an increase in the quality of private external child care also leads to an increase in the price for child care, parents will increase their demand for children and at the same time increase their labor supply if the price effect is not too large.

Regarding privately provided external child care, our results for households opting for publicly provided child care predict that there is a crowding-out effect with respect to all
policy instruments for secondary earners with a relatively high wage rate. As mentioned above, only an increase in the quality of publicly provided child care may have a positive effect on the secondary earner’s labor supply while for all other instruments the effect is negative. Parents thus do not use the publicly provided child care to substitute parental child care time but rather substitute private external child care time. For households opting for a parental child care benefit, we observe a crowding-out effect of private external child care for all three instruments influencing publicly provided child care. For these households, only an increase in the parental child care benefit may lead to a stronger demand for private external child care but at the same time it has a negative effect on the secondary earner’s labor supply. This result is similar to Havnes and Mogstad (2011) who evaluate the effect of new subsidized child care on labor supply. They find that the subsidized child care mostly crowds out informal care arrangements implying a significant net cost of the child care arrangement.

Our results with respect to the child’s quality suggest that for low income households with a large parental child care quality opting for public child care, an increase in both the quantity and quality of as well as in the subsidy for publicly provided child care is likely to have a positive effect. These households benefit from redistribution via the income tax. The same applies to households opting for a parental child care benefit with respect to an increase in the parental child care benefit. For high income households, however, the impact of the policy instruments is likely to be negative due to the negative income effect.

In case of households choosing an interior solution of parental and external child care, we find that both policy instruments, a parental child care benefit and publicly provided child care, are equally effective with respect to welfare and the household’s decision which instrument to choose only depends on the relative benefit the household receives in total. This decision is independent of the household’s income. For households choosing a corner solution, on the other hand, the decision depends on a variety of parameters: their income, their preferences for children, child care and consumption as well as on the policy parameters. Low income households choosing a corner solution of staying at home and not consuming external child care opt for a parental child care benefit if they have a small preference for children and their quality. For high income households, the decision depends on qualitative differences between private and public external child care. If the relative benefit the households receive from the policy instruments is identical and the quality of private external child care is larger than the quality of publicly provided child care, households in this corner solution also benefit more from the parental child care benefit than from publicly provided child care.
References


Mathematical Appendix

A: First-order conditions of the maximization problem

The first-order conditions of the household’s maximization problem in (4.4) are the following:

\[
\frac{\partial U_i}{\partial n_i} = \frac{\Psi_i - \gamma_i(Y_i + w_i)(1 - t)}{n_i \Psi_i - (Y_i + w_i)(1 - t)} \tag{A.1}
\]

\[
\frac{\partial U_i}{\partial h_i} = \frac{\gamma_i \rho_i q_h}{1 + h_i q_h} - \frac{(1 - \gamma_i) n_i w_i (1 - t)}{(Y_i + w_i)(1 - t) - \Psi_i} \tag{A.2}
\]

\[
\frac{\partial U_i}{\partial s_i} = \frac{\gamma_i (1 - \rho_i) q_s}{1 + s_i q_s + \alpha_i g q_g} - \frac{(1 - \gamma_i) n_i \pi_s}{(Y_i + w_i)(1 - t) - \Psi_i} \tag{A.3}
\]

\[
\frac{\partial U_i}{\partial z_i} = \frac{\gamma_i \eta_i}{z_i} - \frac{(1 - \gamma_i) n_i B}{(Y_i + w_i)(1 - t) - \Psi_i} \tag{A.4}
\]

with

\[
\Psi_i \equiv n_i \pi_s + h_i w_i (1 - t) + z_i B + \alpha_i g \pi_g (1 - \beta) - (1 - \alpha_i) \sigma
\]

B: Demand functions

Demand function for parental child care in the model:

\[
h_i^* = \begin{cases} 
1 & \text{if } w_i < \tilde{w}_i^h \text{ and } n_i < 1 \\
\frac{1}{n_i} & \text{if } w_i < \tilde{w}_i^h \text{ and } n_i \geq 1 \\
\frac{1}{n_i} & \text{if } \tilde{w}_i^h \leq w_i \leq \tilde{w}_i^h \text{ and } n_i \left(\frac{\rho_i P_{n_i} q_h - \eta_i w_i (1 - t)}{\eta_i w_i q_h (1 - t)}\right) \geq 1 \\
\frac{\rho_i P_{n_i} q_h - \eta_i w_i (1 - t)}{\eta_i w_i q_h (1 - t)} & \text{if } \tilde{w}_i^h \leq w_i \leq \tilde{w}_i^h \text{ and } n_i \left(\frac{\rho_i P_{n_i} q_h - \eta_i w_i (1 - t)}{\eta_i w_i q_h (1 - t)}\right) < 1 \\
0 & \text{if } w_i > \tilde{w}_i^h 
\end{cases} \tag{4.B.1}
\]
Demand function for external child care in the model:

\[ s_i^* = \begin{cases} 
0 & \text{if } w_i < \hat{w}_i^s \\
\frac{(1 - \rho_i)P_{n,i}q_s - \eta_s \pi_s (1 + \alpha_i g q_g)}{\eta_s \pi_s q_s} & \text{if } \hat{w}_i^s \leq w_i \leq \hat{w}_u^s \\
1 - \alpha_i g & \text{if } w_i > \hat{w}_u^s 
\end{cases} \]  

(4.B.2)

4.C: Comparative statics results: Interior solution

The results for changes in the secondary earner's wage rate, parental child care quality and the quality of privately provided external child care on fertility, parental child care, the demand for private external child care and the child's quality are the following:

\[ \frac{\partial n_i^*}{\partial w_i} = -\gamma \eta_i [(1 - t) - \frac{\pi_s - \alpha_i g \pi_g - \pi_g (1 - \beta) q_s}{q_s} - (1 - \alpha_i) \sigma] \]  

(C.1)

\[ \frac{\partial n_i^*}{\partial q_h} = \frac{\gamma \eta_i (Y_i + w_i) w_i (1 - t)^2}{(P_{n,i})^2 (q_h)^2} \]  

(C.2)

\[ \frac{\partial h_i^*}{\partial w_i} = -\frac{\rho_i \left[ P_{n,i} - \frac{w_i (1 - t)}{q_h} \pi_s \right]}{\eta_i (w_i)^2 (1 - t)} \]  

(C.3)

\[ \frac{\partial h_i^*}{\partial q_h} = \frac{\eta_i - \rho_i}{\eta_i (q_h)^2} \]  

(C.4)

\[ \frac{\partial s_i^*}{\partial w_i} = \frac{(1 - \rho_i)(1 - t)}{\eta_i \pi_s q_h} \]  

(C.5)

\[ \frac{\partial s_i^*}{\partial q_h} = -\frac{(1 - \rho_i)w_i (1 - t)}{\eta_i \pi_s (q_h)^2} \]  

(C.6)

\[ \frac{\partial q_i^*}{\partial w_i} = \frac{\phi_i q_s}{w_i q_h} \left[ \rho_i P_{n,i} q_h + (1 + \eta_i) w_i (1 - t) \right] \]  

(C.7)

\[ \frac{\partial q_i^*}{\partial q_h} = -\frac{\phi_i q_s}{(q_h)^2} \left[ \rho_i P_{n,i} q_h + (1 + \eta_i) w_i (1 - t) \right] \]  

(C.8)

The results for the comparative statics analysis for child-specific consumption are the following:
\[ \frac{\partial z \ast_i}{\partial w_i} = \frac{(1 - t)}{Bq_h} \]  
\[ \frac{\partial z \ast_i}{\partial q_h} = -\frac{w_i(1 - t)}{B(q_h)^2} \] \[ \frac{\partial z \ast_i}{\partial q_s} = -\frac{(1 - \kappa)\pi_s(1 + \alpha_i g q_g)}{B(q_s)^2} \] \[ \frac{\partial z \ast_i}{\partial q_g} = g\{\alpha_i \bar{m} q_h[\pi_s q_g - \kappa \pi_g(1 - \beta)q_s] - \kappa \alpha \beta \bar{n} w_i \pi_g q_s\} \] \[ \frac{\partial z \ast_i}{\partial g} = \frac{\alpha_i \bar{m} q_h[\pi_s q_g - \pi_g(1 - \beta)q_s] - \alpha \beta \bar{n} w_i \pi_g q_s}{\bar{m} B q_h q_s} \] \[ \frac{\partial z \ast_i}{\partial \beta} = \frac{\alpha_i \bar{m} q_h - \alpha \bar{n} w_i}{\bar{m} B q_h} \] \[ \frac{\partial z \ast_i}{\partial \sigma} = \frac{(1 - \alpha_i)\bar{m} q_h - (1 - \bar{\alpha})\bar{n} w_i}{\bar{m} B q_h} \]

D: Comparative statics results: Corner solutions

The results for changes in the secondary earner’s wage rate and parental child care on the child’s quality for the corner solutions are the following:

\[ \frac{\partial q \ast_i}{\partial q_h} \bigg|_{h_i=0,q_i=1} = \frac{-\phi_i|_{h_i=0,q_i=1} (1 + \alpha_i g q_g)}{\rho_i P_{n,i}(q_h)^2 (1 + q_h)} \left[ \eta_i w_i (1 - t)(1 + q_h) - \rho_i P_{n,i}(q_h)^2 \right] \] \[ \frac{\partial q \ast_i}{\partial w_i} \bigg|_{h_i=0,q_i=1} = \frac{-\eta_i \phi_i|_{h_i=0,q_i=1} - \alpha_i g q_g}{P_{n,i} q_h} \left[ 1 + q_s (1 - \alpha_i g) + \alpha_i g q_g \right] (1 - t) \]
The results for the comparative statics analysis for the corner solution where the secondary earner decides to stay at home full-time with the children with $n_i > 1$ are the following:

\[
\frac{\partial q_i^*|_{h_i=1, s_i=0}}{\partial q_i} = \phi_i|_{h_i=1, s_i=0} g \left\{ \alpha_i (1 - \rho_i) + \frac{\eta_i (1 + \alpha_i g q_i)}{m P_{n_i, q_h}} \left[ (\pi_s q_g - \pi_g (1 - \beta) s) - \beta \bar{w} q_g q_s \right] + \frac{\rho_i (1 + \alpha_i g q_i) \left[ \alpha_i \bar{m} \pi_s q_g - \bar{m} \pi_g (1 - \beta) q_s \right] + \beta \bar{w} q_g q_s \left[ \bar{a} \pi_g + \bar{a} \pi_s + \alpha_i g \left[ \pi_s q_g - \pi_g (1 - \beta) q_s \right] + \sigma q_s \right]}{P_{n_i, q_h + \gamma_i} (Y_i + w_i) (1 - t)} \right\}
\]

(D.3)

\[
\frac{\partial q_i^*|_{h_i=1, s_i=0}}{\partial g} = \phi_i|_{h_i=1, s_i=0} g \left\{ \alpha_i (1 - \rho_i) + \frac{\eta_i (1 + \alpha_i g q_i)}{m P_{n_i, q_h}} \left[ (\pi_s q_g - \pi_g (1 - \beta) s) - \beta \bar{w} q_g q_s \right] + \frac{\rho_i (1 + \alpha_i g q_i) \left[ \alpha_i \bar{m} \pi_s q_g - \bar{m} \pi_g (1 - \beta) q_s \right] + \beta \bar{w} q_g q_s \left[ \bar{a} \pi_g + \bar{a} \pi_s + \alpha_i g \left[ \pi_s q_g - \pi_g (1 - \beta) q_s \right] + \sigma q_s \right]}{P_{n_i, q_h + \gamma_i} (Y_i + w_i) (1 - t)} \right\}
\]

(D.4)

\[
\frac{\partial q_i^*|_{h_i=1, s_i=0}}{\partial \beta} = \phi_i|_{h_i=1, s_i=0} g \left\{ \alpha_i (1 - \rho_i) + \frac{\eta_i (1 + \alpha_i g q_i)}{m P_{n_i, q_h}} \left[ (\pi_s q_g - \pi_g (1 - \beta) s) - \beta \bar{w} q_g q_s \right] + \frac{\rho_i (1 + \alpha_i g q_i) \left[ \alpha_i \bar{m} \pi_s q_g - \bar{m} \pi_g (1 - \beta) q_s \right] + \beta \bar{w} q_g q_s \left[ \bar{a} \pi_g + \bar{a} \pi_s + \alpha_i g \left[ \pi_s q_g - \pi_g (1 - \beta) q_s \right] + \sigma q_s \right]}{P_{n_i, q_h + \gamma_i} (Y_i + w_i) (1 - t)} \right\}
\]

(D.5)

\[
\frac{\partial q_i^*|_{h_i=1, s_i=0}}{\partial \sigma} = \phi_i|_{h_i=1, s_i=0} g \left\{ \alpha_i (1 - \rho_i) + \frac{\eta_i (1 + \alpha_i g q_i)}{m P_{n_i, q_h}} \left[ (\pi_s q_g - \pi_g (1 - \beta) s) - \beta \bar{w} q_g q_s \right] + \frac{\rho_i (1 + \alpha_i g q_i) \left[ \alpha_i \bar{m} \pi_s q_g - \bar{m} \pi_g (1 - \beta) q_s \right] + \beta \bar{w} q_g q_s \left[ \bar{a} \pi_g + \bar{a} \pi_s + \alpha_i g \left[ \pi_s q_g - \pi_g (1 - \beta) q_s \right] + \sigma q_s \right]}{P_{n_i, q_h + \gamma_i} (Y_i + w_i) (1 - t)} \right\}
\]

(D.6)

with $\phi_i|_{h_i=1, s_i=0} \equiv \theta \left( \frac{P_{n_i, q_h}}{B} \right)^m \left( \frac{\gamma_i \eta_i (Y_i + w_i) (1 - t) + \gamma_i \eta_i g (Y_i + w_i) (1 - t)}{\gamma_i \eta_i (1 + \alpha_i g q_i) (Y_i + w_i) (1 - t)} \right)^{\beta_i}$.