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Optimal Public Provision of Nursing Homes and the Role of Information *

Michael Kuhn† Robert Nuscheler‡

31 May 2007

Abstract

Increasing demand for long-term care poses at least four challenges to the policy-maker: (i) How should care be supplied, within a nursing home or within the family? (ii) What level of care should be provided in the different arrangements? (iii) How do the answers relate to the severity of dependence? (iv) How can financial strain be mitigated for families with severely dependent members? The problems are aggravated when individual severity is the family's private information. We consider a theoretical model of long-term care provision under adverse selection. Households who are assumed to be altruistic towards dependent members decide on the amount of care and on the context of provision: within the household or within a nursing home. Nursing homes provide more effective care for severe cases but impose a disutility from being institutionalized on all cases. The regulator sets a transfer to redistribute consumption and, where relevant, to finance public nursing homes. We derive the allocations under full and asymmetric information with and without nursing homes, respectively, and examine under which conditions nursing homes improve social welfare. Our main result is that by imposing a utility loss without offering greater effectiveness in the care for mildly dependent cases, the nursing home facilitates self-selection and mitigates and possibly eliminates distortions in caring levels and transfers. Informational asymmetries may thus lead to care being provided too often within institutions rather than within a family context.

Keywords: adverse selection, long-term care, nursing homes, redistribution

JEL classification numbers: D82, H23, I18, J14.

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1 Introduction

In recent years population ageing has moved into a top place of the policy agenda of most industrialized countries. As more and more people live to a high age, the ailments and frailty coming with oldest age are going to increase the demand for long-term care at population level even though it is recognized that most of the life years gained are healthy years.[1] It is therefore generally accepted that social expenditure for long-term care is going to remain significant.[2] In Germany, for instance, long-term care demand is expected to triple by 2050 as compared to the 1999 level (Schultz et al. 2001). To make ends meet there will be more nursing homes and more alternative care arrangements, including family care. The latter not only introduces a considerable psychological strain on the care giver but also a financial burden when the care giver has to compromise on labour market participation.[3] The increasing demand for long-term care thus poses at least four challenges to the policy-maker: (i) How should care be supplied, within a nursing home or within the family? (ii) What level of care should be provided in the different arrangements? (iii) How do the answers relate to the severity of dependence? (iv) How can financial strain be mitigated for families with severely dependent members?

Issues (i) and (ii) receive already a considerable amount of attention in the policy arena. It is generally recognized that a trend towards a more flexible provision of care within a household or community context is welcome and, as levels of disability are declining, also feasible. Lakdawalla and Philipson (2002) argue that the market for long-term care is prone not only to exhibit an increase in demand but also an increasing supply as the

[1]It is expected that as an OECD average one in four persons will belong to the group 65+ by the year 2040. At the same time the share of the oldest old (80+) within this group is going to increase from around one in five to one in three (OECD 2005).

[2]Current (year 2000) spending levels within OECD countries range between 0.2 per cent and 3 percent of GDP with the bulk lying between 0.5 per cent and 1.6 per cent of GDP (OECD 2005). Public expenditure on long-term care amounts to a share of 10 to 20 per cent of public health care spending.

[3]In a survey conducted in Germany 83 per cent of care givers reported a high or very high perceived burden of their activity. Moreover, 30 per cent declared that they reduced labour market participation (Schneekloth and Müller 2000).

younger old take over, at the point of retirement, caring responsibilities for the oldest old. Indeed, the supply of informal care explains very well the decrease in per capita output within the US market for long-term care over the period 1971-95. This notwithstanding, the market for nursing home care will continue to exist at significant levels as the scope for within family provision of care is limited both by the degree of disability and by the time constraints faced by informal carers. The mix and the interplay between formal and informal care and the context of provision - within the household or within a nursing home - is going to remain on the agenda. With regard to issue (iv), policy-makers in a number of countries have come to recognize the financial strain on families who care for dependent members and seek to mitigate it by reimbursing informal carers for private expenditure and time allocated to the provision of care.[4] Where families with severely dependent members receive considerable financial transfers problems are prone to arise whenever the degree of dependency is the family's private information. It is easy to envisage that families try to exaggerate the degree of dependency and their efforts in the provision of care in order to become eligible for higher benefits. While there are a number of mechanisms for the policy-maker to reveal the degree of dependency, the inefficiencies related to them place a bound on the scope of redistribution and generally lead to a distorted allocation of care. Combining issues (i) to (iv) raises the question under which form of provision the informational problems are greater or, in other words, whether one form of provision turns out to be superior on informational grounds.

We consider a theoretical model of care giving under adverse selection.[5] Households are assumed to be altruistic towards their dependent members. Nursing homes provide more effective care for severe cases but impose a disutility from being institutionalized on all cases. The regulator sets a transfer to redistribute consumption and, where relevant,

[4]These benefits may come both in cash or in-kind. See OECD (2005: Table 1.1) for an overview of different public long-term care programmes.

[5]We should stress that we do not consider adverse selection within an ex-ante insurance context, where the propensity to become severely dependent is private information. We rather consider a context of ex-post moral hazard, where individuals have an incentive to misreport the degree of (realized) dependency in order to receive greater benefits. In the concluding section 5 we comment on how our model can be read in the context of long-term-care insurance.

to finance public nursing homes. We derive the allocations under full and asymmetric information with and without nursing homes, respectively, and examine under which conditions nursing homes improve social welfare. Under complete information the regulator fully compensates for the consumption loss suffered by families providing care for severe cases. Institutional care for severe cases increases social welfare if and only if its effectiveness is sufficiently large relative to the disutility from institutionalization. Asymmetric information generally leads to distorted caring levels and transfers both with and without nursing homes and limits redistribution towards families with severely dependent members. By imposing a utility loss without offering more effective care for mildly dependent cases, the nursing home facilitates self-selection and possibly eliminates distortions in caring levels and transfers. We thereby provide a rationale based on informational asymmetries for why care may be provided too often within institutions rather than within a family context.

Our model relates to two strands of literature. First, it contributes to the (theoretical) economics of long-term care.[6] Our work is closest in spirit to Pestieau and Sato (2004) and Jousten et al. (2005). Pestieau and Sato (2004) consider the mix in the provision of formal and informal care to dependent parents when their children differ in their productivity in the labour market. The study develops an optimal policy comprising the public provision of a nursing home, a subsidy paid to children providing informal care and a flat tax on earnings. While the model thus addresses issues of the right mix in the allocation of care under redistributive concerns it is set out under complete information only. Jousten et al. (2005) deals with the allocation of care within the family or within a nursing home when children differ in the degree of altruism towards their parents. While they study the optimal policy (transfer/provision of public nursing home) under asymmetric information as we do, their model differs in a number of respects: First, the adverse selection problem arises with respect to the degree of the children's altruism (perfect or not there at all); second their assumptions about the technology of the nursing home technology make it always inferior to the provision of care within the family; third, in their model the nursing home is always provided (only the level of care provided within the nursing home

[6]See Norton (2000) for an overview on the economics of long-term care.

is endogenous), whereas in our model not only the care levels are endogenous but also the technology under which they are provided (i.e., within the household or in a nursing home). Corresponding to these differences in the set-up our results are rather different. For instance, Jousten et al. (2005) find that the level (quality) of nursing home care is distorted downwards in order to make the nursing home an unattractive option for altruistic children who would provide care for their parents themselves but may be willing to send them to a nursing home if this guarantees them a financial transfer. In our model, the provision of care for severely dependent parents is distorted upwards (within the nursing home or the family) as this makes it a more costly option for the children of only mildly dependent parents.

Second, our work contributes to the literature on the use of public expenditure for redistributive purposes (Nichols and Zeckhauser 1982, Blackorby and Donaldson 1988, Besley and Coate 1991, Boadway and Marchand 1995). This work studies how public expenditure has to be structured in order to allow for redistribution towards the poor subject to self-selection constraints. In Besley and Coate (1991) self-selection is achieved through quality reductions in publicly provided private goods (where the rich prefer a higher quality). In Boadway and Marchand (1995), individuals seek to attain an optimal provision by amending a public provision by private purchases. Here, the over-provision of the publicly provided good fully crowds out the private provision of a rich individual who then selects pure private provision. Our model goes beyond this literature in that it distinguishes the *level* of provision from the *technology* of provision. Technology is relevant in the following sense: For any level of care, provision within the nursing home is more effective for highly dependent types; while a nursing home leads to a direct utility loss. As it turns out, the planner facilitates self-selection (and redistribution) not only through distortions in the levels of provision (as was hitherto known) but also through distortions in the technology choice. In particular, a desire to reduce informational rents may lead to the choice of an ineffective or even 'hurtful' technology. The latter corresponds to a 'self-selection through ordeal' argument as was proposed but not formalized by Nichols and Zeckhauser (1982).

The remainder of the paper is structured as follows. Section 2 introduces the model and Section 3 characterizes the first-best provision for the family and nursing home, respectively, and provides a condition under which public nursing homes should be introduced. Section 4 provides the solution under asymmetric information leading up to our main result regarding the differences in the provision of nursing homes under first and second-best circumstances. We discuss a number of extensions in Section 5 and offer concluding remarks in Section 6. Some of the more technical proofs and all figures are relegated to the appendix.

2 The model

We consider a model economy where an individual cares for a disabled relative. To fix ideas we suppose a setting with two generations where one parent has one child.[7] While the children are (potential) care givers parents are considered to have problems with activities of daily living (ADLs), i.e. they are in need of care.[8] In general there are three different forms of care: (i) care provided within a family context (informal or *family care*), (ii) care purchased on the market (*market care*), and (iii) inpatient care provided by a nursing home (*nursing home care*). For the purpose of the current paper we will not distinguish between family and market care but concentrate on the peculiarities of nursing home care and its implications for public policy.

There are two severity types, H (high) and L (low), and the share of high severity parents is denoted $h \in (0,1)$. Let $a \geq 0$ denote the level of care or *attention* a parent receives measured in money. Then utility of the parent derived from family or market care is given by

$$v_i^F(a) = v_i(a), \tag{1}$$

where the subscript $i \in \{L, H\}$ refers to the severity type and the superscript F to family

[7] We thereby abstract from strategic considerations like the demonstration effect in populations of three generations as suggested by Cox and Stark 2005 and from private provision games between children as highlighted by Konrad et al. 2002.

[8] Other obvious settings include an individual caring for her or his spouse or a parent caring for a disabled child.

care. As usual we let $v_i' > 0$ and $v_i'' < 0$. The following assumptions are crucial:

$$\forall a \geq 0 : v_L(a) > v_H(a), \tag{2}$$

$$\forall a \geq 0 : v_L'(a) < v_H'(a). \tag{3}$$

While equation (2) simply is the definition of high severity, equation (3) suggests that attention or care is more productive when the parent in need is a high severity type: an additional unit help is, at all care levels, more valuable for parents having major problems in performing ADLs.

If care is provided in a nursing home, the parent utility is

$$v_H^N(n) = \mu v_H(n) - \overline{v}, \tag{4}$$

$$v_L^N(n) = v_L(n) - \overline{v}, \tag{5}$$

where $n \geq 0$ is the amount per patient spent by the nursing home. The parameter $\overline{v} \geq 0$ captures the disutility parents suffer when being moved from their accustomed environment to an institution.[9] We consider this loss to be independent from severity. In contrast, the effectiveness or productivity of nursing home care hinges on severity: for any given level of care provided to H-types, the nursing home is at least as productive as family care, i.e. $\mu \geq 1$. Whereas the nursing home has no productivity advantage for L-types. Thus, from a parent perspective, low severity types should never be taken care of in a nursing home. The overall costs of nursing home care are θn, where $\theta \in [0, 1]$ is the share of parents that are taken care of in a nursing home. We consider that nursing homes are publicly financed by (optimal) income taxes.

The utility of the child is given by

$$U_i = u(c) + \beta v_i(a). \tag{6}$$

We consider that all children are identical and use all their time to generate labour market income $y > 0$. When they provide care, a, reduced labour market participation lets income

[9] When parents were living with their children, $\overline{v}$, can be understood as the dread from separation. However, in many issues, family or ambulatory care is provided to a parent who is still living in their own property. In this case $\overline{v}$ is the disutility from giving up their 'own place'.

drop to $y - a$.[10] The disposable income (consumption) then is $c = y - a - T$, where T is a tax (subsidy) paid to (by) the government.[11] The consumption utility is given by the first term of equation (6). As is standard we assume

$$u' = \frac{du}{dc} > 0 \text{ and } u'' = \frac{d^2u}{dc^2} < 0.$$

The second term of equation (6) captures that children typically care about the wellbeing of their parents. The parameter β will in general lie between 0 and 1. To avoid a conflict of interest between children and the social planner we consider children to be perfect altruists, i.e. $\beta = 1$.[12] Note that our model then has an alternative interpretation with u as the consumption utility of the "parent" and a the monetary resources (s)he allocates to care. These resources may buy care on the private market (a is formal care) or may be a transfer to the child (bequest) that stimulates attention (a is informal care). Thus, our model is more general than it may at first appear. In particular, when adopting the alternative interpretation, altruism is not necessary for our results.

For $\beta = 1$ the social welfare function can be written as

$$W = hU_H + (1 - h)U_L. \tag{7}$$

Note that this formulation does not imply that we do not count the utility of the parents. It simply means that we avoid double counting of the parents' utility.[13] The social planner now maximizes (7) subject to the budget constraint

$$hT_H + (1 - h)T_L = \theta n. \tag{8}$$

With type-dependent income taxes and type-contingent nursing home care provision, however, the planner's ability to directly observe severity levels is crucial. In the following

[10]The assumption that children's time use is restricted to work and care only is crucial for our model. In Section 5 we discuss the changes when leisure is introduced as an additional possibility.

[11]Note that a may alternatively interpreted as care provided by health care professionals (market care): the child still earns income y and pays a for market care.

[12]Jousten et al. (2005) consider the degree of altruism β to be private information of the child. Using an optimal taxation model they derive the second-best optimal long-term care insurance.

[13]Jousten at al. 2005, for instance, make the same assumption. See Hammond 1987 for a justification.

Section 3 severity is considered observable and the solution to the maximization problem will be referred to as first-best. In Section 4 severity is private information of the family. Type contingent contracts thus have to be incentive compatible and a second-best optimal long-term care arrangement results.

3 First-best long-term care

We start out by deriving the optimal long-term care arrangement without nursing homes ($\theta = 0$). We then derive the optimal policy with nursing homes when all high severity parents are allocated to a nursing home ($\theta = h$). Finally, we ask whether and when the provision of nursing homes is efficient.

3.1 The optimal policy without nursing homes

The social planner maximizes the objective function W_F, given in equation (7), with respect to a_H, a_L, T_H, and T_L subject to the budget constraint $hT_H + (1-h)T_L = 0$. The index 'F' indicates that we are in the case with family care only. Using the budget constraint to substitute for T_H we get

$$W_F = h\left(u\left(y - a_H + \frac{1-h}{h}T_L\right) + v_H(a_H)\right) + (1-h)\left(u(y - a_L - T_L) + v_L(a_L)\right). \quad (9)$$

The first-order conditions are

$$\frac{\partial W_F}{\partial a_H} = h\left(-u'_H + v'_H\right) = 0,$$
$$\frac{\partial W_F}{\partial a_L} = (1-h)\left(-u'_L + v'_L\right) = 0,$$
$$\frac{\partial W_F}{\partial T_L} = (1-h)u'_H - (1-h)u'_L = 0,$$

with $u'_i := u'(c_i)$. This is the standard optimal income taxation result that can be summarized as

$$v'_H(a^F_H) = u'(c^F_H) = u'(c^F_L) = v'_L(a^F_L), \quad (10)$$

where the super-script 'F' is used to denote the first-best values with family care only.

Lemma 1 *The first-best policy without nursing homes has the following properties:*

(i) $a_H^F > a_L^F$,

(ii) $c_H^F = c_L^F = y - ha_H^F - (1-h)\, a_L^F$ *and*

(iii) $T_L^F = h\left(a_H^F - a_L^F\right) > 0 > (1-h)\left(a_L^F - a_H^F\right) = T_H^F$.

The results of the lemma are quite intuitive: The first result (i) simply states that more family care is provided, when care is more productive, i.e. when H-types need assistance. Like in optimal direct taxation models with perfect information, the utilitarian social planner eliminates income inequality and, as shown in (ii), identical consumption levels obtain. Obviously, income equalization with type-dependent care levels can only be achieved through redistributive taxation, where children of H-types receive a net transfer and L-types have to pay taxes. Figure 1 illustrates.

[Figure 1 about here]

To see the structural equivalence to the standard optimal direct taxation framework (e.g. Stiglitz 1982), consider that y is the time endowment of an individual. If the individual refrains from supplying labour it receives utility $v_i(y)$. With labour supply c_i, however, the individual gets $u(c_i) + v_i(y - c_i)$. Let $a_i \equiv y - c_i$ then we are in the framework considered here, though, the interpretation is different. Note that in the optimal direct taxation interpretation of the model the L-types are the more productive individuals since they can provide an additional unit labour at lower cost than H-types, $v_L' < v_H'$.

3.2 The optimal policy with nursing homes

As argued above, given the utility of L-types from nursing home care (5), it cannot be efficient to allocate low severity patients to a nursing home. With perfect information we can at most have $\theta = h$. Using (1), (4), (6) and substituting $T_H = n - \frac{1-h}{h} T_L$ from the budget constraint the social objective is

$$W_N = h\left(u(y + \frac{1-h}{h} T_L - n) + \mu v_H\left(n\right) - \overline{v}\right) + (1-h)\left(u(y - a_L - T_L) + v_L(a_L)\right), \quad (11)$$

where the index 'N' is used to denote the case with nursing homes for the H-types and family care for the L-types.

The first-order conditions are

$$\frac{\partial W_N}{\partial n} = h\left(-u'_H + \mu v'_H\right) = 0, \tag{12}$$
$$\frac{\partial W_N}{\partial a_L} = (1-h)\left(-u'_L + v'_L\right) = 0, \tag{13}$$
$$\frac{\partial W_N}{\partial T_L} = (1-h)u'_H - (1-h)u'_L = 0, \tag{14}$$

implying

$$\mu v'_H(n^N) = u'(c_H^N) = u'(c_L^N) = v'_L(a_L^N), \tag{15}$$

where we use the super-script 'N' to denote the first-best variables when H-types are allocated to a nursing home.

Lemma 2 *If H-types are allocated to a nursing home and if $\mu > 1$, then the optimal policy has the following properties:*

(i) $n^N > a_H^F > a_L^F > a_L^N$,

(ii) $c_H^N = c_L^N = y - hn^N - (1-h)\, a_L^N$,

(iii) $T_H^N = hn^N + (1-h)\, a_L^N > h\left(n^N - a_L^N\right) = T_L^N > 0$.

(iv) *For $\mu = 1$, we have $n^N = a_H^F > a_L^F = a_L^N$ and $c_H^N = c_L^N = c_H^F = c_L^F$.*

Again, the results are intuitive. (i) Due to the higher productivity of nursing homes as compared to the family, H-types receive more care with nursing homes than without. In turn, this implies a higher care level for H-types than for L-types. (ii) The social planner continues to equalize consumption levels. But since nursing homes improve the care technology the overall resources spent on care are higher and, consequently, the consumption levels are lower than without nursing home care. (iii) An immediate implication from (ii) is that the taxes to be paid by L-types increase. Note that H-types now pay taxes rather than receiving a transfer. The children of the severe cases do not provide any care and generate income y. Their tax payment may therefore be interpreted as a co-payment for the nursing home care of the elderly. Figure 2 illustrates the changes that are triggered by μ. With our multiplicative technology specification the marginal utility of care for high severity types turns upwards where the root of v'_H is the pivotal point. With our assumptions on utility functions, equation (15) describes a horizontal line that is strictly above the one following from equation (10) yielding the results of the lemma.

[Figure 2 about here]

3.3 First-best public provision of nursing homes

Let W_N^* and W_F^* denote the maximized welfare functions with and without nursing home care respectively. The social welfare gain achieved by the introduction of publicly provided nursing homes is then given by $\Delta := W_N^* - W_F^*$. First-best nursing home provision is then characterized by the following simple decision rule: provide nursing home care to H-types if and only if $\Delta \geq 0$. Using (9) and (11) and observing that consumption is equalized across types independent of whether nursing home care is available or not, we can write

$$\Delta(\overline{v}, \mu) = \begin{array}{l} u(c_H^N) - u(c_H^F) + h\left[\mu v_H\left(n^N\right) - \overline{v} - v_H\left(a_H^F\right)\right] \\ +(1-h)\left[v_L\left(a_L^N\right) - v_L\left(a_L^F\right)\right]. \end{array} \tag{16}$$

The following lemma provides a characterization of the function $\Delta(\overline{v}, \mu)$ (see Appendix for the proof).

Lemma 3 *The function $\Delta(\overline{v}, \mu)$ has the following properties:*

(i) $\frac{d\Delta}{d\overline{v}} = -h < 0,$

(ii) $\frac{d\Delta}{d\mu} = hv_H\left(n^N\right) > 0,$

(iii) $\lim\limits_{\overline{v}\to\infty} \Delta(\overline{v}, \mu) = -\infty,$

(iv) $\lim\limits_{\mu\to\infty} \Delta(\overline{v}, \mu) = \infty$

(v) $\Delta(0, \mu) > (=)0$ *if* $\mu > (=)1,$

(vi) $\frac{d\overline{v}}{d\mu}|_{\Delta=0} = v_H\left(n^N\right) > 0,$

(vii) $\frac{d^2\overline{v}}{d\mu^2}|_{\Delta=0} = v'_H\left(n^N\right)\frac{dn^N}{d\mu} > 0$

The first five properties are rather intuitive. When the disutility of nursing home care increases, the provision of nursing homes becomes less desirable (i). This holds also in the limit where costs are prohibitive (iii). If there is no disutility, however, social welfare can be increased whenever nursing homes improve the productivity of care, i.e. when $\mu > 1$ (v). The social planner is indifferent between nursing homes and family care when $\overline{v} = 0$ and $\mu = 1$. An increase in the productivity of nursing homes makes them socially more desirable (ii), and again, this holds in the limit (iv).

The social planner is indifferent between family care only and nursing home provision for H-types whenever $\Delta = 0$. For every $\mu \geq 1$ define

$$\overline{v}(\mu) := \arg_{\overline{v}}\{\Delta(\overline{v}, \mu) = 0\}. \tag{17}$$

Strict monotonicity of Δ in its arguments guarantees that $\overline{v}(\mu)$ is a singleton. Thus, $\overline{v} : [1, \infty) \to [0, \infty)$, $\mu \mapsto \overline{v}(\mu)$ is a function that is, due to the properties (vi) and (vii), increasing and convex in μ.[14]

Proposition 1 *The function $\overline{v}(\mu)$ defines a locus in the nursing home technology space $(\overline{v}, \mu)$ such that $\Delta = 0$ on the locus, $\Delta < 0$ for all pairs $(\overline{v}, \mu)$ with $\overline{v} > \overline{v}(\mu)$ and $\Delta > 0$ for all pairs $(\overline{v}, \mu)$ with $\overline{v} < \overline{v}(\mu)$.*

The proof follows directly from the properties of $\Delta(\overline{v}, \mu)$ and $\overline{v}(\mu)$ given in Lemma 3. This proposition tells us that the public provision of nursing homes is efficient whenever the productivity gain, $\mu - 1$, is large enough as compared to the disutility of being institutionalized, $\overline{v}$. This result is illustrated in Figure 3 below where nursing home provision is efficient to the south-east of the locus $\Delta = 0$ (the shaded area) and inefficient to the north-west of $\Delta = 0$.

[Figure 3 about here]

[14]Note that whenever we write $\overline{v}(\mu)$ we refer to the function just defined. We use $\overline{v}$ *without* any argument for the disutility parameter.

4 Optimal care under asymmetric information

Implementation of the first-best policy studied above typically requires that the social planner can observe the severity levels of the parents. Apart from participation constraints the planner can then freely set type contingent transfers or taxes, care levels and dictate the care technology to be used by the respective types. In reality severity is not readily observable. Although the planner may obtain some information on severity, e.g. by conducting costly severity audits, we restrict ourselves to the case where severity is private information of the family.[15] But then the planner has to obey incentive constraints when deriving the optimal, second-best, policy. As will turn out the cases with and without nursing homes differ substantially.

4.1 The second-best without nursing homes

With family care only the optimal policy has to obey the following incentive compatibility (IC) constraints

$$u(y - a_H - T_H) + v_H(a_H) \geq u(y - a_L - T_L) + v_H(a_L) \tag{ICH}$$

$$u(y - a_L - T_L) + v_L(a_L) \geq u(y - a_H - T_H) + v_L(a_H). \tag{ICL}$$

for the H-type and L-type, respectively. When the first-best levels of care ($a_H^F > a_L^F$) and consumption ($c_H^F = c_L^F$) are considered, we find that (ICL) is violated, whereas (ICH) holds as a strict inequality. From (ICH) and (ICL) we obtain the monotonicity (M) condition

$$[v_H(a_H) - v_H(a_L)] - [v_L(a_H) - v_L(a_L)] \geq 0 \Leftrightarrow a_H - a_L \geq 0, \tag{M}$$

where the equivalence follows from our assumption $v'_H > v'_L$. It will turn out that (M) will generally be satisfied for our problem. The social planner maximizes (9) subject to the budget constraint $hT_H + (1 - h)T_L = 0$, (ICH) and (ICL). Denoting by ψ_H and ψ_L, with $\psi_H \geq 0; \psi_L \geq 0$, the Langrangean multiplier of (ICH) and (ICL), respectively, and substituting for $T_H = -\frac{1-h}{h}T_L$ we obtain the following set of first-order conditions for a_H,

[15] In Section 5 we discuss the changes when a costly severity audit is available.

a_L and T_L.

$$(h + \psi_H) (-u'_H + v'_H (a_H)) - \psi_L (-u'_H + v'_L (a_H)) = 0, \tag{18}$$

$$(1 - h + \psi_L) (-u'_L + v'_L (a_L)) - \psi_H (-u'_L + v'_H (a_L)) = 0, \tag{19}$$

$$(1 - h) (u'_H - u'_L) + (\psi_H - \psi_L) \left(\frac{1-h}{h} u'_H + u'_L \right) = 0. \tag{20}$$

Denoting second-best variables with a $\widehat{\cdot}$ and continuing to use 'F' for the case in which care is provided within the family only, we can characterize the solution as follows (see Appendix for the proof).

Lemma 4 *When care is provided within the family, the second-best allocation under asymmetric information has the following properties:*

(i) *General structure: The second-best allocation involves* $\widehat{\psi}_L > \widehat{\psi}_H = 0; \widehat{c}_H^F < \widehat{c}_L^F$ *and* $\widehat{a}_H^F > \widehat{a}_L^F$.

(ii) *Distortions:* $\widehat{a}_L^F$ *is conditionally efficient, i.e. satisfies* $u'_L \left(\widehat{c}_L^F \right) = v'_L \left(\widehat{a}_L^F \right)$ *and* $\widehat{a}_H^F$ *is distorted upwards, i.e. satisfies* $u'_H \left(\widehat{c}_H^F \right) > v'_H \left(\widehat{a}_H^F \right)$.

Under adverse selection, the planner has to concede an information rent to L-types in order to induce them to reveal truthfully their identity. The planner faces the standard rent extraction efficiency trade off: the contract for the L-types is typically improved as compared to the first-best contract and the contract for the H-types is made less attractive from the perspective of an L-type. Lemma 4 tells us that this trade off results in conditional efficiency for the L-types and in an upward distortion of care levels for the H-types. This drives a wedge between the consumption levels allocated to the two types, where L-types are allowed a greater consumption. Thus, as is common in models of optimal income taxation, asymmetric information reduces the scope for redistribution.[16]

[16]Note that either effort a_i or gross income $y - a_i$ is observable/contractible. The second option concurs with the income taxation literature. However, strictly speaking we would have the choice of leisure as a further confounding variable. The first option may appear to be at odds with the usual assumption of unobservable effort. Yet, in our model, observable effort does not relax the information problem (about severity). For instance, in Germany transfers to the family for the provision/purchase of home care are linked to a measure of the hours of care provided/purchased.

It is difficult to make any statements as to how the second-best levels of care and consumption compare to the first-best. Intuitively, one would expect perhaps that the transfer paid by the L-type is lowered, such that $\widehat{T}_L^F < T_L^F$ and, thus, $\widehat{c}_L^F > c_L^F = c_H^F > \widehat{c}_H^F$.[17] This intuitive solution is illustrated in Figure 4. Note that the incentive constraint for the L-types is binding in optimum. This implies that the hatched area in the figure (loss of consumption utility) is exactly as large as the shaded area (gain in care utility). The figure also suggests that the care level for H-types under asymmetric information ($\widehat{a}_H^F$) is higher than in the first-best solution (a_H^F). But this does not hold in general. While $\widehat{c}_H^F < c_H^F$ would suggest $\widehat{a}_H^F < a_H^F$, the upward distortion for efficiency reasons may imply the opposite. But then it cannot be ruled out that extreme distortions in $\widehat{a}_H^F$ overturn the 'intuitive' solution. For instance, for $\widehat{a}_H^F \gg a_H^F$, it is possible that $\widehat{T}_L^F > T_L^F$ so that $\widehat{c}_H^F < \widehat{c}_L^F < c_L^F$. Here, H-types are forced to provide so much (excessive) care that the transfer paid by the L-types is increased. Consequently consumption is lower for both types. Alternatively, for $\widehat{a}_H^F < a_H^F$ we may find $c_H^F < \widehat{c}_H^F < \widehat{c}_L^F$. In this case, care is under-provided to H-patients but the scope for consumption increases for both types.[18]

[Figure 4 about here]

4.2 The second-best with nursing homes

We now turn to the remaining case, where care is provided within a nursing home under asymmetric information about severity. Recall that it is suboptimal to accommodate L types in the nursing home. Thus, assuming that the nursing home cannot turn down patients, the following IC constraints must hold

$$u(y - T_H) + \mu v_H(n) - \overline{v} \geq u(y - a_L - T_L) + v_H(a_L), \qquad \text{(ICHN)}$$

$$u(y - a_L - T_L) + v_L(a_L) \geq u(y - T_H) + v_L(n) - \overline{v}. \qquad \text{(ICLN)}$$

[17] This would also imply $\widehat{a}_L^F > a_L^F$.

[18] As the comparison between the second-best and first-best levels is not substantive for our subsequent analysis, we do not provide a formal derivation of these results.

The constraint (ICHN) must hold as the planner cannot force even severe types into a nursing home. (ICHN) and (ICLN) imply the monotonicity condition

$$\mu v_H(n) - v_H(a_L) \geq v_L(n) - v_L(a_L), \tag{MN}$$

which is satisfied if $n \geq a_L$.

The social planner maximizes (11) subject to the budget constraint $h\,(T_H - n) + (1 - h)T_L = 0$, (ICHN) and (ICLN). Continuing to use $\psi_i, i = H, L$ as Langrangean multiplier on constraint (ICiN), the first-order conditions for n, a_L and T_L are given by

$$(h + \psi_H)\left(-u'_H + \mu v'_H(n)\right) - \psi_L\left(-u'_H + v'_L(n)\right) = 0, \tag{21}$$

for n and by (19) and (20) for a_L and T_L, respectively. The situation turns out to be more complex than in the case without nursing homes. This is because the direct utility loss from nursing care, $\overline{v}$, may induce countervailing incentives in the following sense.[19] If $\overline{v}$ is low the situation is similar to the case without nursing homes, where L-types have an incentive to mimic H-types in order to obtain a greater financial transfer and a greater care level. However, in contrast to the family setting, posing as an H-type now comes with a direct utility loss for the parent when being institutionalized. This relaxes (ICLN), and for intermediate values of $\overline{v}$ both (ICLN) and (ICHN) may be slack. However, if $\overline{v}$ grows large enough then (ICHN) binds. Here, children of H-types have to be given an incentive to send their parents to a nursing home despite the direct disutility $\overline{v}$. The following lemma provides a more formal characterization of the three regimes.

Lemma 5 *Consider the first-best allocation* $\left\{n^N, a_L^N, c_H^N, c_L^N\right\}$. *The second-best allocation then involves*

(i) *Regime 1 with* $\widehat{\psi}_L > \widehat{\psi}_H = 0$ *if and only if* $\overline{v} < v_L(n^N) - v_L(a_L^N)$,

(ii) *Regime 2 with* $\widehat{\psi}_L = \widehat{\psi}_H = 0$ *if and only if* $\overline{v} \in \left[v_L(n^N) - v_L(a_L^N), \mu v_H(n^N) - v_H(a_L^N)\right]$,

(iii) *Regime 3 with* $0 = \widehat{\psi}_L < \widehat{\psi}_H$ *if and only if* $\overline{v} > \mu v_H(n^N) - v_H(a_L^N)$.

[19]See Laffont and Martimort (2002: section 3.1) on countervailing incentives.

Proof. Note that $n^N > a_L^N$ so that (MN) is satisfied and the interval referred to in part (ii) of the lemma is non-empty. The lemma follows immediately from inserting the inequalities in (i) and (iii) into (ICHN) and (ICLN), respectively. ■

The second-best allocation within each of the regimes is then characterized as follows.

Lemma 6 *When H-types are cared for in a nursing home, the second-best allocation under asymmetric information has the following properties:*

- *Regime 1:* (i) $\widehat{c}_H^N < \widehat{c}_L^N$*;* (ii) $\widehat{a}_L^N$ *conditionally efficient; and* (iii) $\widehat{n}^N$ *distorted upwards.*

- *Regime 2:* $\widehat{c}_i^N = c_i^N$, $\widehat{a}_L^N = a_L^N$, $\widehat{n}^N = n^N$, *i.e. the first-best levels.*

- *Regime 3:* (i) $\widehat{c}_H^N > \widehat{c}_L^N$; (ii) $\widehat{n}^N$ *conditionally efficient; and* (iii) $\widehat{a}_L^N$ *could be distorted upwards or downwards.*

Proof. Regime 1: As $\widehat{\psi}_L > \widehat{\psi}_H = 0$ the proof follows in analogy to the proof of part (ii) of Lemma 4. Regime 2: For $\widehat{\psi}_L = \widehat{\psi}_H = 0$ the first-best levels in care and consumption are realized. Regime 3: Here, $0 = \widehat{\psi}_L < \widehat{\psi}_H$. It then follows from (20) that $\widehat{c}_H^N > \widehat{c}_L^N$. Conditional efficiency of $\widehat{n}^N$ follows from (21), where $\widehat{\psi}_L = 0$. Finally, using $\widehat{\psi}_L = 0$ in (19) we obtain $(1-h)\left(u_L' - v_L'(a_L)\right) = \psi_H\left(u_L' - v_H'(a_L)\right)$. Noting that $u_L' - \mu v_H'(n) = u_L' - u_H' > 0$ and that $v_H'(a_L) \lesseqgtr \mu v_H'(n)$ depends on μ, we see that neither $u_L' > \max\left\{v_H'(a_L), \mu v_H'(n)\right\}$ nor $v_H'(a_L) > u_L' > \mu v_H'(n)$ can be ruled out. ■

The allocation thus depends on the direct disutility of parents sent to a nursing home, $\overline{v}$. For low levels of $\overline{v}$ (regime 1) the allocation is similar to the one without nursing homes. Note, however, the upward distortion in the provision of nursing care. This stands in contrast to the finding by Jousten et al. (2005), where care is under-provided in the nursing home. Like for the case of family care, we cannot make statements about the deviation of the second-best choices from their first-best values. In particular, we cannot assess whether $\widehat{n}^N$ lies above or below the first-best level n^N. Whereas rent extraction calls for higher care levels under asymmetric information, $\widehat{n}^N < n^N$ cannot be ruled out. This situation may arise if the marginal utility of nursing care for the L-type, $v_L'\left(\widehat{n}^N\right)$ is relatively high as compared to the marginal utility of consumption u_H'. In this case, there is not much scope to extract rents by way of over-provision of care. The excess provision of nursing care required to allow for more redistribution may then turn out to be so high that both types lose out on consumption. In this case, it may be more effective to under-provide care $\widehat{n}^N \ll n^N$ and, thereby, grant more consumption to both types.

For high levels of $\overline{v}$ (regime 3) the incentive problem is reversed, where the children of H types would have to be given an incentive to send their parents to a nursing home. This is done by granting them a higher level of consumption (i.e. by reducing the tax/fee for nursing care) and allocating an efficient quality of nursing care. Furthermore, family care is rendered unattractive for H types by imposing a distortion in the level of care that is subsidized. The direction of the distortion depends on the preferences of the H-type. If for the level of consumption $\widehat{c}_L^N$ granted to H-children, when mimicking an L type, H-parents still have a high propensity to benefit even from family care, i.e. if $v'_H\left(\widehat{a}_L^N\right) > u'_L(\widehat{c}_L^N)$, then the level of family care, $\widehat{a}_L^N$, is distorted downwards from its efficient level. The converse is true if H-parents do not stand to gain too much from family care, i.e. if $v'_H\left(\widehat{a}_L^N\right) < u'_L(\widehat{c}_L^N)$.

Notably, for intermediate levels of $\overline{v}$ (regime 2) the first-best *levels* in care and consumption are attainable. As they do not benefit from the greater effectiveness of the nursing home, L types stand to benefit less from a nursing home than H types. But then if the disutility from nursing care $\overline{v}$ is sufficiently large but not too large, the nursing home becomes an unattractive option to L types, while retaining attraction for H types. Here, natural separation is feasible.

4.3 Second-best public provision of nursing homes

Proceeding along similar lines as for the first best, let $\widehat{W}_F$ and $\widehat{W}_N$ denote the maximized second-best welfare functions with and without nursing homes respectively. The net welfare gain from the introduction of nursing homes in a second-best environment is then given by $\widehat{\Delta} := \widehat{W}_N - \widehat{W}_F$, where nursing homes should be introduced to provide care for

H types if and only if $\widehat{\Delta} \geq 0$. Inserting the respective second-best variables we can write

$$\widehat{\Delta}(\overline{v}, \mu) = h\left[u(\widehat{c}_H^N) + \mu v_H\left(\widehat{n}^N\right) - \overline{v} - u(\widehat{c}_H^F) - v_H\left(\widehat{a}_H^F\right)\right] \\ + (1-h)\left[u(\widehat{c}_L^N) + v_L\left(\widehat{a}_L^N\right) - u(\widehat{c}_L^F) - v_L\left(\widehat{a}_L^F\right)\right]. \quad (22)$$

Note that due to informational rents, there is no longer an equalization of consumption across types. The second best-allocation$\left\{\widehat{n}^N, \widehat{a}_L^N, \widehat{c}_H^N, \widehat{c}_L^N\right\}$ depends on both $\overline{v}$ and μ in a non-trivial way. Furthermore, the precise relationships change across regimes 1-3. Thus, it is difficult to characterize the locus for which $\widehat{\Delta}(\overline{v}, \mu) = 0$ and compare it to the locus $\Delta(\overline{v}, \mu) = 0$, where the planner is indifferent in the first-best between whether care is provided within the household or within an institution. In order to gain some leeway, we draw on a graphical representation in $(\mu, \overline{v})$ space.

In Lemma 3 and Figure 3 we already characterized the locus $\Delta = 0$ but how does this locus relate to the three regimes? Noting that the allocation in the first-best does not depend on $\overline{v}$ so that $\frac{dn^N}{d\overline{v}} = \frac{da_L^N}{d\overline{v}} = 0$ we can make use of the following definitions:

$$\overline{v}^-(\mu) := v_L(n^N) - v_L(a_L^N), \quad (23)$$

$$\overline{v}^+(\mu) := \mu v_H(n^N) - v_H(a_L^N). \quad (24)$$

Then equation (23) defines the boundary between regimes 1 and 2 and equation (24) the boundary between regimes 2 and 3. The following lemma provides a further characterization of the boundaries $\overline{v}^-(\mu)$ and $\overline{v}^+(\mu)$ (see Appendix for the proof).

Lemma 7

(i) $\frac{\partial \overline{v}^-}{\partial \mu} = v_L'\left(n^N\right)\frac{dn^N}{d\mu} - v_L'\left(a_L^N\right)\frac{da_L^N}{d\mu} > 0$ *and* $\frac{\partial \overline{v}^+}{\partial \mu} = v_H\left(n^N\right) + \mu v_H'\left(n^N\right)\frac{dn^N}{d\mu} - v_H'\left(a_L^N\right)\frac{da_L^N}{d\mu} > 0$.

(ii) $\overline{v}^+(\mu) > \max\left\{\overline{v}(\mu), \overline{v}^-(\mu)\right\}$ *for all* μ.

(iii) *If* $v_i''' \leq 0$, *then there exists a unique* $\mu^* \in (1, \infty)$ *such that* $\overline{v}^-(\mu) \geq \overline{v}(\mu) \Leftrightarrow \mu \leq \mu^*$.

The lemma confirms the clear-cut ordering of regimes 1-3 in $(\mu, \overline{v})$ space: For a given μ we have regime 1 for $\overline{v} < \overline{v}^+(\mu)$, regime 2 for $\overline{v} \in \left[\overline{v}^-(\mu), \overline{v}^+(\mu)\right]$ and regime 3 for

$\overline{v} > \overline{v}^{-}(\mu)$. More importantly, it establishes that the $\Delta = 0$ locus, $\overline{v}(\mu)$, crosses $\overline{v}^{-}(\mu)$ only once and from below and never crosses $\overline{v}^{+}(\mu)$. Figure 5 below illustrates the three regimes (panel A) and how the first-best locus $\Delta = 0$ relates to them (panel B). The dark shaded area in Panel B identifies the nursing home technologies that result in regime 2 *and* in nursing home care under perfect information. Since it is possible to implement efficient care levels and efficient transfers intuition suggests that, in regime 2, nursing home care under symmetric information implies nursing home care under asymmetric information. This intuition will be confirmed below. Whenever regime 3 is realized in the second-best, a nursing home cannot be optimal from a first-best perspective. Indeed, we will show below that regime 3 never arises even in a second-best context. It is thus convenient to characterize the $\widehat{\Delta}(\overline{v}, \mu) = 0$ locus for regimes 2 and 1 and to ignore regime 3.

[Figure 5 about here]

Lemma 8 *Consider regimes 1 and 2. The function* $\widehat{\Delta}(\overline{v}, \mu)$ *has the following properties*

(i) $\frac{d\widehat{\Delta}}{d\overline{v}} = \frac{dV(\overline{v},\mu)}{d\overline{v}} = -\left(h - \widehat{\psi}_L\right) < 0,$

(ii) $\frac{d\widehat{\Delta}}{d\mu} = \frac{dV(\overline{v},\mu)}{d\mu} = hv_H\left(\widehat{n}^N\right) > 0,$

(iii) $\lim_{\overline{v}\to\infty} \widehat{\Delta}(\overline{v}, \mu) = -\infty,$

(iv) $\lim_{\mu\to\infty} \widehat{\Delta}(\overline{v}, \mu) = \infty$

(v) $\widehat{\Delta}(0, \mu) > (=)0$ *if* $\mu > (=)1,$

(vi) $\frac{d\overline{v}}{d\mu}\Big|_{\widehat{\Delta}=0} = \frac{hv_H\left(\widehat{n}^N\right)}{h-\widehat{\psi}_L} > 0.$

Proof. In order to show (i) and (ii) we recall that $\widehat{W}_F$ is constant in $(\overline{v}, \mu)$ and make use of the value function

$$V(\overline{v}, \mu) = \widehat{W}_N(\overline{v}, \mu) + \widehat{\psi}_L\left[u(y - \widehat{a}_L^N - \widehat{T}_L^N) + v_L(\widehat{a}_L^N) - u(y - \widehat{T}_H^N) - v_L(\widehat{n}^N) + \overline{v}\right],$$

with $\widehat{\psi}_L > 0$ in regime 1 and $\widehat{\psi}_L = 0$ in regime 2. We can then write $\widehat{\Delta}(\overline{v}, \mu) = V(\overline{v}, \mu) - \widehat{W}_F$ and, using the envelope theorem, we obtain the derivatives stated in (i) and (ii).

To show that $-(h-\widehat{\psi}_L)<0$ in part (i) rearrange (20), with $\widehat{\psi}_H=0$, to obtain $\widehat{\psi}_L=\frac{(1-h)\left(u'_H-u'_L\right)}{(1-h)u'_H/h+u'_L}$. But then $h-\widehat{\psi}_L=\frac{u'_L}{(1-h)u'_H/h+u'_L}>0$. Parts (ii)-(vi) follow in analogy to the proof of the corresponding parts in Lemma 3. ■

The properties are similar to those established with regard to $\Delta(\overline{v},\mu)$ in Lemma 3. From part (v) we note that similar to the first-best, the planner is indifferent between a nursing home and a family setting for $\overline{v}=0$ and $\mu=1$. This is because the allocations in the nursing home and family setting are equivalent, both involving precisely the same distortions. For every $\mu\geq 1$ define

$$\widehat{\overline{v}}(\mu):=\arg_{\overline{v}}\left\{\widehat{\Delta}(\overline{v},\mu)=0\right\}. \tag{25}$$

Strict monotonicity of $\widehat{\Delta}$ in its arguments guarantees that $\widehat{\overline{v}}(\mu)$ is a singleton. Thus, $\widehat{\overline{v}}:[1,\infty)\rightarrow[0,\infty)$, $\mu\mapsto\widehat{\overline{v}}(\mu)$ is a function that is, due to property (vi), increasing in μ. For regime 1 we also note that in the limit $(\overline{v},\mu)\rightarrow(\overline{v}^-(\mu),\mu)$ we have $\widehat{\psi}_L\rightarrow 0$ and $\widehat{x}^N\rightarrow x^N$ with $x\in\{c_L,c_H,a_L,n\}$. Thus, $\widehat{\overline{v}}(\mu)$ is continuous at $\widehat{\overline{v}}(\mu)=\overline{v}^-(\mu)$ should this exist. Similar to the first-best case we then obtain the following proposition.

Proposition 2 *The function $\widehat{\overline{v}}(\mu)$ defines a locus in the nursing home technology space $(\overline{v},\mu)$ such that $\widehat{\Delta}=0$ on the locus, $\widehat{\Delta}<0$ for all pairs $(\overline{v},\mu)$ with $\overline{v}>\widehat{\overline{v}}(\mu)$ and $\widehat{\Delta}>0$ for all pairs $(\overline{v},\mu)$ with $\overline{v}<\widehat{\overline{v}}(\mu)$.*

As in the case of the first-best, the public provision of nursing homes is efficient in a second-best context if and only if the productivity gain, $\mu-1$, is large enough as compared to the disutility of being institutionalized, $\overline{v}$. This does not answer yet the more interesting questions as to how the second-best locus $\widehat{\overline{v}}(\mu)$ relates to the first-best locus $\overline{v}(\mu)$ and what this tells us about discrepancies in the provision of nursing homes that arise from the informational asymmetry. In order to do so, we examine in turn regime 2 and then regime 1.

Consider thus a pair $(\mu,\overline{v})$, satisfying $\overline{v}\in[\overline{v}^-(\mu),\overline{v}^+(\mu)]$ implying that $(\mu,\overline{v})$ gives rise to regime 2. Furthermore, assume $\mu\geq\mu^*$, implying from Lemma 7 that $\overline{v}(\mu)\geq\overline{v}^-(\mu)$. Finally, define $\Delta_F:=\left(W_F^*-\widehat{W}_F\right)$. Note that $\Delta_F>0$ by definition of first- and second-best welfare and that Δ_F is a constant in $(\mu,\overline{v})$-space because both of its components relate to the provision within the family. We can then establish the following

Lemma 9 *Let $\mu \geq \mu^*$ Then $\widehat{\overline{v}}(\mu) - \overline{v}(\mu) = \Delta_F h^{-1} > 0$ is a constant for all μ.*

Proof. Consider $\Delta(\overline{v}, \mu) = (W_N^* - W_F^*) = 0$. This is equivalent to $h\overline{v} = (W_N^* + h\overline{v} - W_F^*)$ or $\overline{v} = (W_N^* + h\overline{v} - W_F^*) h^{-1}$. By construction $\overline{v} = \overline{v}(\mu)$ so that we can write $\overline{v}(\mu) = (W_N^* + h\overline{v} - W_F^*) h^{-1}$.[20] Similarly we can write $\widehat{\overline{v}}(\mu) := \left(\widehat{W}_N + h\overline{v} - \widehat{W}_F\right) h^{-}1$. Then we find $\widehat{\overline{v}}(\mu) - \overline{v}(\mu) = \left[\left(\widehat{W}_N - W_N^*\right) - \left(\widehat{W}_F - W_F^*\right)\right] h^{-}1 = \left(W_F^* - \widehat{W}_F\right) h^{-}1 = \Delta_F h^{-}1 > 0$ for all μ. Here, the second equality follows from the fact that with nursing homes the first-best allocation is realized within regime 2 so that $\widehat{W}_N = W_N^*$. ■

Hence, it follows that within regime 2, the locus for $\widehat{\overline{v}}(\mu)$ lies strictly above the locus $\overline{v}(\mu)$. The distance $\Delta_F h^{-1}$ between the two schedules is determined by the extent of the informational inefficiency in the family context.[21] In the region between $\widehat{\overline{v}}(\mu)$ and $\overline{v}(\mu)$ it is thus efficient to provide nursing homes in a second-best but not in a first-best context – there is an *over-provision* of nursing home care. The reason is that admission to a nursing home allows separation of types and thereby first-best care levels and first-best redistribution. Note that 'only' the levels are first-best efficient. The technology choice is inefficient: the disutility $\overline{v}$ is only incurred under asymmetric information. By continuity the over-provision result should extend to some $\mu \leq \mu^*$.

We turn now to regime 1, i.e. we consider $(\mu, \overline{v})$ that satisfy $\overline{v} \leq \overline{v}^-(\mu)$. From parts (v) of Lemmas 3 and 8 we know that $\widehat{\overline{v}}(1) = \overline{v}(1) = 0$, hence the two schedules coincide at the origin. Whereas we also know the slopes $\frac{\partial \widehat{\overline{v}}}{\partial \mu} = \frac{h v_H\left(\widehat{n}^N\right)}{h - \widehat{\psi}_L}$ and $\frac{\partial \overline{v}}{\partial \mu} = v_H\left(n^N\right)$ (from parts (vi) of Lemmas 3 and 8) it is difficult to compare them because generally $\widehat{n}^N \neq n^N$. Nonetheless, we can pin down a few more characteristics of $\widehat{\overline{v}}(\mu)$ (see Appendix for the proof).

Lemma 10

(i) *Let $v_i''' \leq 0$. Then there exists a unique $\mu^{**} \in (1, \mu^*)$ such that $\overline{v}^-(\mu) \geq \widehat{\overline{v}}(\mu) \Leftrightarrow \mu \leq \mu^{**}$.*

[20] This expression for $\overline{v}(\mu)$ is obviously just implicit as the RHS depends on $\overline{v}$. We use the expression for mathematical expedience but caution not to interpret it as a closed form for $\overline{v}(\mu)$.

[21] Note that the lemma also implies that $\frac{\partial \overline{v}}{\partial \mu} = \frac{\partial \widehat{\overline{v}}}{\partial \mu}$ holds in regime 2. This is readily verified from parts (vi) of Lemmas 3 and 8, respectively, when observing that $\widehat{n}^N = n^N$ and $\widehat{\psi}_L = 0$ in regime 2.

(ii) *There exists a* $\mu^{***} \in [1, \mu^{**})$ *such that* $\widehat{\overline{v}}(\mu) > \overline{v}(\mu)$ *if* $\mu > \mu^{***}$.

We have now established the position of the $\widehat{\overline{v}}(\mu)$ locus in $(\mu, \overline{v})$ space as far as we possibly can. Figure 6 illustrates the relevant cases.

[Figure 6 about here]

Specifically, we have established that the $\widehat{\overline{v}}(\mu)$ schedule crosses regime 1 for $\mu \in (1, \mu^{**})$ and that it continues to lie above the $\overline{v}(\mu)$ locus at least for a range of intermediate $\mu \in [\mu^{***}, \mu^{**}]$. These may include all $\mu > \mu^{***} = 1$, as depicted in Panel A of Figure 6. A sufficient condition for this to happen is that $\frac{\partial \widehat{\overline{v}}}{\partial \mu} = \frac{h v_H (\widehat{n}^N)}{h - \widehat{\psi}_L} \geq v_H (n^N) = \frac{\partial \overline{v}}{\partial \mu}$ for all $\mu \in [1, \mu^{**})$. It is readily verified that this is always true if $\widehat{n}^N \geq n^N$. However, for $\widehat{n}^N < n^N$ we cannot rule out that $\overline{v}(\mu)$ and $\widehat{\overline{v}}(\mu)$ intersect for some $\mu \in (1, \mu^{**})$. This case is depicted in Panel B.[22] The following proposition summarizes our main result.

Proposition 3

(i) *There exists a* $\mu^{***} \geq 1$ *such that for all* $\mu > \mu^{***}$ *and for all* $\overline{v} \in (\overline{v}(\mu), \widehat{\overline{v}}(\mu)]$ *public provision of nursing homes is optimal under asymmetric information but not under symmetric information.*

(ii) *The reverse case where public provision of nursing homes is optimal under symmetric information but not under asymmetric information can arise only if* $\widehat{n}^N < n^N$, $\mu < \mu^{***}$ *and* $\overline{v} \in [\widehat{\overline{v}}(\mu), \overline{v}(\mu))$.

Consider Panel A of Figure 6. The shaded area identifies nursing home technologies under which an over-provision of nursing homes occurs under asymmetric information. The technology choice is inefficient in the sense that nursing home care is offered under asymmetric information where it would not be an efficient mode of care under complete information. For $\mu \geq \mu^*$ the intuition is straightforward: the disutility of nursing home care facilitates self selection and thereby enables the planner to implement efficient care

[22] In fact, there may be more than one point of intersection.

levels *and* efficient transfers (regime 2). This works as long as the disutility $\overline{v}$ does not grow too large. For $\mu < \mu^{**}$ the shaded area bears regime 1. Given the productivity of nursing homes μ the disutility $\overline{v}$ is too small for natural separation. Nevertheless, $\overline{v}$ relaxes the binding incentive constraint (ICLN) and thereby allows a greater extent of redistribution and possibly smaller distortion in the nursing homes' care levels. Note that we here have over-provision along two dimensions: nursing homes are provided too often and, when provided care is distorted upwards. For $\mu \in [\mu^{**}, \mu^{*})$ the value of $\overline{v}$ determines whether we are in regime 1 or 2.

If $\widehat{n}^N \geq n^N$ we will always be in a situation as depicted by Panel A, i.e. in a situation with a tendency towards over-provision of and/or within nursing homes. We cannot rule out, however, a scenario as depicted in Panel B. For the shaded area, the productivity of nursing homes is low and the level of nursing care is reduced under asymmetric information below the first-best, i.e. $\widehat{n}^N < n^N$. In such a case, nursing homes are not publicly provided under asymmetric information where they would be under symmetric information. For a sharp reduction in nursing care, the productivity advantage of nursing home care diminishes by so much that it becomes unattractive in a second-best setting despite the greater scope for redistribution.

5 Discussion

Endogenous nursing home technology choice. We assumed throughout that the nursing home technology is exogenously given. We characterized the parameter ranges in which a nursing home is a favourable option under complete and under imperfect information. It may be more realistic to consider an environment where the planner cannot only decide on the provision of nursing homes and on the care level offered therein but also influence the productivity, μ, and the disutility, $\overline{v}$. To fix ideas think of μ as being the 'medical' quality of a nursing home; the training of staff and medical equipment being important dimensions. From its minimum (which is 1), μ can be increased at some positive cost $c_\mu(\mu)$. Maybe the most suitable interpretation of $\overline{v}$ is the 'hotel' standard of the nursing home. From its initial value, which is assumed to be strictly positive, the nursing home can reduce the disutility by improving, say, the room standard or the catering quality. This comes at a cost $c_{\overline{v}}(\overline{v})$. As usual, we assume that $c'_\mu > 0$, $c''_\mu > 0$, $c'_{\overline{v}} < 0$, and $c''_{\overline{v}} > 0$. With an endogenous nursing home technology, one may end up with an institution that is close to a hospital (high μ but also a high $\overline{v}$), a fancy retirement resort (low $\overline{v}$ but also low μ) or with an institution with less extreme characteristics. More interesting, however, are the marginal incentives of the social planner and, in particular, how these incentives differ between a first-best and a second-best environment. Lemma 3 tells us how social welfare changes in the first-best optimum when (i) $\overline{v}$ and (ii) μ change. Unsurprisingly, social welfare increases when $\overline{v}$ drops and increases as μ increases. Now let both, $\overline{v}$ and μ be optimally chosen, then the marginal social gain of each investment is just equal to its marginal social cost. Lemma 8 tells us how the marginal investments incentives under asymmetric information compare to those of Lemma 3. Comparison of parts (ii) of the respective lemmas tells us that, again, the result hinges on whether or not the second-best nursing home care level is above or below its first-best level. As already argued above, we cannot make general statements on this. Things change when we compare parts (i) of the two lemmas. When we are in regime 2, both incentive constraints are slack ($\widehat{\psi}_L = \widehat{\psi}_H = 0$) and the marginal incentives to invest in nursing home hotel quality is independent on severity information. Things change when regime 1 is considered ($\widehat{\psi}_L > 0$). While in the first-best the marginal incentive to invest in $\overline{v}$ is given by h, we have $(h - \widehat{\psi}_L) \in (0, h)$ in the second-best. With convex costs we clearly have a *higher* disutility under asymmetric information than under complete information. This is intuitively appealing since the disutility plays an important role in the incentives constraints: the higher the disutility (ignoring regime 3) the less demanding the incentive constraints and the greater the scope for redistributive transfers. Anecdotal evidence suggests that the main concern about nursing homes is the low quality of life offered therein (high $\overline{v}$) rather than poor medical quality. Our framework offers an information based rationale for why the optimal policy may result in lower quality than would be desirable in a 'perfect world'.

Nursing home location. Closely related to the endogenous nursing home technology discussion above is location choice. If a nursing home is located in a large city it may be relatively easy to recruit well trained staff. So investments in μ would be relatively cheap as compared to remote locations. The disutility $\overline{v}$ may, for several reasons, be lower in cities. The likelihood that relatives live close to the nursing home, is likely to be higher than otherwise, for instance. Moreover, large cities offer attractions like, museums and opera houses unavailable in remote areas. Thus, a nursing home in a large city is likely to have a high μ and a low $\overline{v}$. While this is certainly good under complete information, self-selection under asymmetric information may require considerable distortions in the care level in order to make the nursing unattractive for L-types. It may therefore be a sensible strategy to establish nursing homes in remote areas where it is potentially much easier to achieve separation. Since $\overline{v}$ has many dimensions one may, of course, also generate a high disutility in large cities. To build a nursing home just besides a highway being only one example.

Modeling nursing home technology. We note that the nursing home technology is biased, in a sense, in favour of the severely dependent H-types. This is because for the utility specification $\mu v_H(n)$ effectiveness, μ, and level of care, n, are technological complements. Thus, the gains from greater effectiveness fully accrue to H-types both through higher μ and through the associated increase in n. L-types tend to lose out as they have to co-finance the greater provision n by way of a higher net transfer to the planner. The question of who stands to benefit from a greater effectiveness may have a bearing, however, on the equilibrium structure. To see this recall that in the presence of nursing care only two regimes could arise under asymmetric information: Regime 1, where (the children of) L-types seek to mimic H-types; and regime 2, where there is natural separation. In principle, a third regime, regime 3, is possible, where the children of H-types have to be given an incentive to send their parents to a nursing home rather than mimic L-types. As it turns out this regime cannot arise as an equilibrium outcome. This is because it corresponds to combinations of low effectiveness and high disutility for which the provision of family care is always optimal even under asymmetric information. However, a conflict of interest between the policy-maker and (the children of) H-types may arise when the benefits of greater effectiveness μ do not fully accrue to H-types. Thus, consider a set-up where for a utility function $v_H(\mu n)$, effectiveness, μ, and level of care, n, are substitutes. For a greater μ a given level of utility $v_H(\mu n)$ could then be attained at lower cost n, allowing a reduction of taxes of both H- and L-types. If the share of L-types is sufficiently large, it cannot be ruled out then that in a first-best setting the planner may wish to introduce nursing homes for H-types even if this is not in their private interest. Regime 3 may then turn up as part of the equilibrium structure.

Long-term care insurance. We cast our argument in a model of tax financed long-term care. While this corresponds to the institutional set-up of a number of countries (e.g. Norway, Spain, Sweden or the UK), other countries rely at least partially on long-term care insurance (e.g. Germany, Japan, Switzerland or the USA). Our model can be interpreted in the context of long-term care insurance when we adopt the following interpretation. Assume that for an elderly person there are only two states: L and H. At the point of signing the insurance contract the probability of becoming severely dependent, h, is identical for everyone. Thus, the premium is the same for everyone and amounts to $\overline{T} = ha_H + (1-h)\,a_L$ without nursing homes and $\overline{T} = hn + (1-h)\,a_L$ with nursing homes, respectively. Under complete information about severity, insurance benefits would then be given by $B_i = a_i$, $i = H, L$, in the absence of nursing homes, and $B_L = a_L$ and $B_H = 0$ if nursing home care is offered free of charge.[23] The adverse selection problem then arises ex-post, when it comes to assigning insurance benefits according to unobservable severity. Here, the benefits have to be designed in a way that rules out misreporting.[24]

Leisure. In footnotes 10 and 16 we briefly discussed the role of leisure. We assumed throughout that children can use their time in only two ways, that is, labour and care. The planner was, thus, always able to infer the actual care levels from observed labour market income. With asymmetric information and family care only the planner separated types

[23] If nursing home care is offered at a fee that covers its marginal cost, the premium is $\overline{T} = (1-h)\,a_L$ and benefits are $B_L = a_L$ and $B_H = n$.

[24] Of course, adverse selection may also arise ex-ante with respect to the remaining healthy and/or unhealthy life-expectancy. Sloan and Norton (1997) provide empirical evidence suggesting that this form of adverse selection is one cause for the low level of private long-term-care insurance in the US.

by distorting care levels upwards. This made the contract for H-types an unattractive option for L-types: The only way to generate low income to qualify for the transfer was to provide the prescribed care level to their parent. But if leisure is an additional option of time use, the child may, at least to some extent, substitute care by leisure reducing the planner's scope for redistributive taxation. Since the planner directly sets care levels of nursing homes this mode of care becomes more attractive when leisure is introduced into the model distorting technology choice even further.

Severity audit. In our model the planner can only learn about the degree of dependency of individuals from the reports made by family members (or, equivalently, on the basis of their choices from the menu of long-term care contracts). In reality, many health care systems rely on the direct verification of dependency levels by experts. In Germany, for instance, a dependency level and the corresponding level of care and transfers are ascribed only after the dependent person has been examined by an expert physician. In the extreme, the planner could always implement the first best if a perfect and costless audit was available. More generally, the availability of an imperfect and/or costly audit will relax the incentive compatibility constraints and thereby mitigate the associated inefficiency.[25] While this argument immediately extends to our set-up, we conjecture that the availability of an imperfect audit would not fundamentally change our main result. Thus, the planner would presumably apply the audit both when care is solely provided within the family *and* when public nursing homes are provided. While the audit would clearly increase the efficiency in both cases, it would turn over the tendency towards over-provision of nursing homes under asymmetric information only if the increase in efficiency was *significantly* greater for an audit performed in a family care context. Since the diagnosing technology available is independent of the mode of care adopted, we do not see reasons for why this should be the case.

[25] See Laffont and Martimort (2002: section 3.6) for a brief exposition of the issue and seminal literature. Cremer et al. (2004) consider the effects of an audit in the determination of disability benefits as a route to (early) retirement.

6 Conclusions

We have derived the allocation of long-term care and redistributive transfers both in the absence and in the presence of nursing homes and both under complete and asymmetric information about the degree of dependency. We have characterized the allocation in terms of the nursing home technology: its effectiveness/productivity of care and the direct utility loss associated with institutional care. Unsurprisingly, nursing homes should be provided publicly if and only if they are sufficiently effective in relation to the loss of utility due to institutionalization. Under asymmetric information this rule becomes biased, however, usually in favour of nursing homes: nursing home care is provided too often and when provided the care level may be distorted upwards. This is because the direct utility loss of nursing care (and the potentially distorted care level) provides a disincentive for the children of less dependent parents to dress them up as severe cases. Thus, informational rents are lower and the scope for redistribution towards families with severely dependent parents is greater. We argued that this may provide the planner with incentives to under-invest in the hotel quality of nursing homes to achieve more favourable selection properties. Only if the level of care offered in nursing homes is severely biased downwards under asymmetric information can a situation arise, where nursing homes are not provided in a second-best where they should in a first-best.

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Appendix – Proofs

Proof of Lemma 3: To begin with note that W_F^* does not depend on $(\overline{v}, \mu)$ as the parameters related to nursing home care are irrelevant in the context of family care. Hence, the following holds.

(i) $\frac{\partial \Delta}{\partial \overline{v}} = -h < 0$ as changes in $\overline{v}$ do not affect any of the optimal choices $\{c_L^N, c_H^N, a_L^N, n^N\}$

(ii) With nursing homes the envelope theorem applies, i.e. $\frac{\partial \Delta}{\partial \mu} = \frac{\partial W_N}{\partial \mu}|_{x=x^N} = hv_H\left(n^N\right) > 0$, where $x \in \{c_L, c_H, a_L, n\}$.

(iii) Follows for $h > 0$ as all choices $\{c_L^N, c_H^N, a_L^N, n^N\}$ and W_F^* are finite.

(iv) From (15) we obtain $\underset{\mu\to\infty}{lim} n^N = \frac{y}{h}$ and $\underset{\mu\to\infty}{lim} a_L^N = \underset{\mu\to\infty}{lim} c_H^N = 0$. This is because in the limit all income is allocated to nursing care. But then, $\underset{\mu\to\infty}{lim} W_N^*(\overline{v}, \mu) = \underset{\mu\to\infty}{lim} h\left(\mu v_H\left(\frac{y}{h}\right) - \overline{v}\right) = \infty$. Since W_F^* is finite we obtain $\underset{\mu\to\infty}{lim} \Delta(\overline{v}, \mu) = \infty$

(v) $\Delta(0, \mu) = W_N^*(0, \mu) - W_F^*(0, \mu) = u(c_H^N) - u(c_H^F) + h\left[\mu v_H\left(n^N\right) - v_H\left(a_H^F\right)\right] + (1-h)\left[v_L\left(a_L^N\right) - v_L\left(a_L^F\right)\right]$
$\geq W_N(0, \mu)\Big|_{(c_L, c_H, a_L, n) = (c_L^F, c_H^F, a_L^F, a_H^F)} - W_F^*(0, \mu) = h(\mu - 1)v_H\left(a_H^F\right) \geq 0$. The first equality is just the definition of Δ, evaluated at $\overline{v} = 0$ and $\mu \geq 1$. The second equation uses the objective function of the respective cases given in equations (9) and (11). The first inequality results since we depart from the optimal values of the endogenous variables in the nursing home case. The rest follows by substitution.

(vi) Total differentiation of equation (16) yields $d\Delta = \frac{\partial \Delta}{\partial \mu}d\mu + \frac{\partial \Delta}{\partial \overline{v}}d\overline{v} = hv_H\left(n^N\right)d\mu - hd\overline{v}$. For $d\Delta = 0$ we obtain the implicit relationship between μ and $\overline{v}$ as given in the lemma.

(vii) Follows from repeated differentiation of $d\Delta$.

Proof of Lemma 4:

(i) We first show by contradiction that an allocation with $\psi_H > 0$ cannot be an equilibrium. To see this, consider first $\psi_H > 0$ and $\psi_L > 0$. As both (ICH) and (ICL) bind, it follows that $v_H(a_H) - v_H(a_L) = v_L(a_H) - v_L(a_L)$. Since $v'_H > v'_L$, the equation can be true if and only if $a_H = a_L = a$. But then from (ICH) and (ICL), $c_H = c_L = c$. The latter implies $u'_H = u'_L = u'$ and thus $\psi_H = \psi_L = \psi$ as from (20). Substituting into (18) and (19) and solving each equation for u' we obtain $u' = v'_H(a) + \frac{\psi}{h}(v'_H(a) - v'_L(a)) = v'_L(a) - \frac{\psi}{1-h}(v'_H(a) - v'_L(a))$. As is readily verified, the second equation implies $v'_H(a) = v'_L(a)$, a contradiction. Next, consider $\psi_H > 0$ and $\psi_L = 0$. Then, from (20), $u'_H < u'_L$ and, thus, $c_H > c_L$. From (ICL), it then follows that $a_H < a_L$. But this contradicts (M). Hence, $\psi_H = 0$. Noting that $\psi_L = \psi_H = 0$ implies $c_H = c_L$ and $a_H > a_L$, which violates (ICL). Therefore, $\widehat{\psi}_L > \widehat{\psi}_H = 0$. Using this in (20), it follows that $u'_H > u'_L$ and, thus, $\widehat{c}^F_H < \widehat{c}^F_L$. In turn, it must be true from (ICH) that $\widehat{a}^F_H > \widehat{a}^F_L$, which completes the proof.

(ii) From (19) we have $u'_L = v'_L(a_L)$ implying conditional efficiency. Furthermore, $h(u'_H - v'_H(a_H)) = \psi_L(u'_H - v'_L(a_H)) \geq \psi_L(u'_H - v'_L(a_L)) = \psi_L(u'_H - u'_L) > 0$, where the first equality follows from (18), the first inequality follows under observation of $\widehat{a}^F_H > \widehat{a}^F_L$ and $v''_L \leq 0$. The second equality follows from (19) and the last inequality follows from $\widehat{c}^F_H < \widehat{c}^F_L$. But then $u'_H > v'_H(a_H)$, implying the upward distortion.

Proof of Lemma 7:

(i) Follows from straightforward differentiation of (23) and (24), respectively, while observing $\frac{dn^N}{d\overline{v}} = \frac{da^N_L}{d\overline{v}} = 0$, $\frac{dn^N}{d\mu} > 0$ and $\frac{da^N_L}{d\mu} < 0$ are obtained from comparative static analysis of the system (12)-(14).

(ii) Recalling $\frac{\partial \overline{v}}{\partial \mu} = v_H\left(n^N\right)$ from property (vi) in Lemma 3 and using the result in part (i) of the present Lemma it is readily checked that $max\left\{\frac{\partial \overline{v}^-}{\partial \mu}, \frac{\partial \overline{v}}{\partial \mu}\right\} < \frac{\partial \overline{v}^+}{\partial \mu}$ for all μ. Furthermore, $\overline{v}^+(1) = v_H(n^N) - v_H(a^N_L) > v_L(n^N) - v_L(a^N_L) = \overline{v}^-(1) > \overline{v}(1) = 0$, where the first inequality follows under observation of $v_H' > v_L'$ and $n^N > a_L^N$. But then, $\overline{v}^+(\mu) > \max\{\overline{v}(\mu), \overline{v}^-(\mu)\}$ for all μ.

(iii) Define $\Delta_{\overline{v}}(\mu) := \overline{v}^-(\mu) - \overline{v}(\mu)$. We seek to show that $\Delta_{\overline{v}}(\mu) \geq 0 \Leftrightarrow \mu \leq \mu^*$. As the continuity of the functions $\overline{v}^-(\mu)$ and $\overline{v}(\mu)$ implies the continuity of $\Delta_{\overline{v}}(\mu)$ it is sufficient to establish (a) $\Delta_{\overline{v}}(1) > 0$; (b) $\lim_{\mu\to\infty} \Delta_{\overline{v}}(\mu) < 0$; and (c) $\Delta_{\overline{v}}(\mu) = 0 \Rightarrow \frac{d\Delta_{\overline{v}}(\mu)}{d\mu} < 0$. (a), (b) and (c) together imply a unique root $\mu^* := \arg_{\mu\in(1,\infty)}\{\Delta_{\overline{v}}(\mu) = 0\}$. (a) has already been established as part of the proof of part (ii). (b) follows as $\lim_{\mu\to\infty} \Delta_{\overline{v}}(\mu) = v_L(\frac{y}{h}) - \lim_{\mu\to\infty} \overline{v}(\mu) < 0$, where $\lim_{\mu\to\infty} \overline{v}(\mu) = \infty$. Here, $\lim_{\mu\to\infty} \overline{v}^-(\mu) = v_L(\frac{y}{h})$ follows from the fact that for $\mu \to \infty$ the whole income is spent on nursing care for the H-types so that $\lim_{\mu\to\infty} n = \frac{y}{h}$. Further, $\lim_{\mu\to\infty} \overline{v}(\mu) = \infty$ follows from the fact that $\lim_{\mu\to\infty} \Delta(\overline{v}, \mu) = \infty = -\lim_{\overline{v}\to\infty} \Delta(\overline{v}, \mu)$ according to properties (iii) and (iv) in Lemma 3. To prove (c), consider

$$\frac{d\Delta_{\overline{v}}(\mu)}{d\mu}\Big|_{\Delta_{\overline{v}}(\mu)=0} = \frac{\partial\overline{v}^-}{\partial\mu}\Big|_{\overline{v}^-(\mu)=\overline{v}(\mu)} - \frac{\partial\overline{v}}{\partial\mu}\Big|_{\overline{v}^-(\mu)=\overline{v}(\mu)}$$
$$= v_L'\left(n^N\right)\frac{dn^N}{d\mu} - v_L'\left(a_L^N\right)\frac{da_L^N}{d\mu} - v_H\left(n^N\right) < 0, \qquad (26)$$

where n^N and a_L^N are the values realized at $(\mu, \overline{v}^-(\mu))$. Since $v_H'' \leq 0$ it follows that $v_H\left(n^N\right) \geq n^N v_H'\left(n^N\right)$. Furthermore, observe $v_L'\left(a_L^N\right) = \mu v_H'\left(n^N\right)$. Hence, it is sufficient for the inequality in (26) that $v_L'\left(n^N\right)\frac{dn^N}{d\mu} - \left[\mu\frac{da_L^N}{d\mu} + n^N\right] v_H'\left(n^N\right) < 0$. Observing $v_L'\left(n^N\right) < v_H'\left(n^N\right)$ it is then sufficient for (26) that $\frac{dn^N}{d\mu} \leq \left[n^N + \mu\frac{da_L^N}{d\mu}\right]$. From comparative statics of the system (12)-(14) we obtain $\frac{dn^N}{d\mu} = \frac{-v_H'\left(n^N\right)A}{\mu v_H'' A + u_H'' u_L'' v_L''} > 0$, where $A := u_L'' v_L'' + \frac{1-h}{h} u_H''\left(u_L'' + v_L''\right) > 0$, and $\frac{da_L^N}{d\mu} = \frac{-v_H'\left(n^N\right)u_H'' u_L''}{\mu v_H'' A + u_H'' u_L'' v_L''} < 0$. Using these expressions one can show after some manipulations that

$$\frac{dn^N}{d\mu} \leq \left[n^N + \mu\frac{da_L^N}{d\mu}\right]$$
$$\Leftrightarrow \left(n^N \mu v_H'' + v_H'\left(n^N\right)\right)A + \left(n^N v_L'' + \mu v_H'\left(n^N\right)\right)u_H'' u_L'' \leq 0.$$

Noting $A > 0$ and $u_H'' u_L'' > 0$ it follows that the second inequality holds if $n^N \mu v_H'' + v_H'\left(n^N\right) \leq 0$ and $n^N v_L'' + \mu v_H'\left(n^N\right) \leq 0$. Recalling $\mu \geq 1$ we have $n^N \mu v_H'' + v_H'\left(n^N\right) \leq n^N v_H'' + v_H'\left(n^N\right) \leq 0$, where the last inequality holds if $v_H''' \leq 0$. Substituting $\mu v_H'\left(n^N\right) = v_L'\left(a_L^N\right)$ and observing $n^N > a_L^N$ we have $n^N v_L'' + \mu v_H'\left(n^N\right) <$

$a_L^N v_L'' + v_L'\left(a_L^N\right) \leq 0$, where the last inequality holds if $v_L''' \leq 0$. Hence, $v_i''' \leq 0$ is sufficient for the inequality in (26). But then, (a)-(c) hold, which implies a unique root $\mu^* := \arg_{\mu\in(1,\infty)}\left\{\Delta_{\overline{v}}(\mu) = 0\right\}.$

Proof of Lemma 10:

(i) The proof is analogous to the proof of part (iii) of Lemma 7. Defining $\Delta_{\widehat{\overline{v}}}(\mu) := \overline{v}^-(\mu) - \widehat{\overline{v}}(\mu)$, we seek to show that $\Delta_{\widehat{\overline{v}}}(\mu) \geq 0 \Leftrightarrow \mu \leq \mu^{**}$. As the continuity of the functions $\overline{v}^-(\mu)$ and $\widehat{\overline{v}}(\mu)$ implies the continuity of $\Delta_{\widehat{\overline{v}}}(\mu)$ it is then sufficient to establish (a) $\Delta_{\widehat{\overline{v}}}(1) > 0$; (b) $\Delta_{\widehat{\overline{v}}}(\mu^*) < 0$; and (c) $\Delta_{\widehat{\overline{v}}}(\mu) = 0 \Rightarrow \frac{d\Delta_{\widehat{\overline{v}}}(\mu)}{d\mu} < 0$. (a), (b) and (c) together imply a unique root $\mu^{**} := \arg_{\mu\in(1,\mu^*)}\left\{\Delta_{\widehat{\overline{v}}}(\mu) = 0\right\}.$

(a) follows as $\widehat{\overline{v}}(1) = 0 < \overline{v}^-(1)$.

(b) We have $\Delta_{\widehat{\overline{v}}}(\mu^*) = \overline{v}^-(\mu^*) - \widehat{\overline{v}}(\mu^*) = -\left[\widehat{\overline{v}}(\mu^*) - \overline{v}(\mu^*)\right] = -\Delta_F h^{-1} < 0$; where the second equality follows as $\overline{v}^-(\mu^*) = \overline{v}(\mu^*)$ by definition of μ^*; and where the third equality and the inequality follow from Lemma 9.

(c) Consider

$$\begin{aligned}
\frac{d\Delta_{\widehat{\overline{v}}}(\mu)}{d\mu}\Big|_{\Delta_{\widehat{\overline{v}}}(\mu)=0} &= \frac{\partial\overline{v}^-}{\partial\mu}\Big|_{\overline{v}^-(\mu)=\widehat{\overline{v}}(\mu)} - \frac{\partial\widehat{\overline{v}}}{\partial\mu}\Big|_{\overline{v}^-(\mu)=\widehat{\overline{v}}(\mu)} \\
&= v_L'\left(n^N\right)\frac{dn^N}{d\mu} - v_L'\left(a_L^N\right)\frac{da_L^N}{d\mu} - \frac{hv_H\left(\widehat{n}^N\right)}{h-\widehat{\psi}_L}\Big|_{\overline{v}^-(\mu)=\widehat{\overline{v}}(\mu)} \\
&= v_L'\left(n^N\right)\frac{dn^N}{d\mu} - v_L'\left(a_L^N\right)\frac{da_L^N}{d\mu} - v_H\left(n^N\right)
\end{aligned}$$

The third equality holds as $\widehat{\psi}_L = 0$ and $\widehat{n}^N = n^N$ for $\widehat{\overline{v}}(\mu) = \overline{v}^-(\mu)$. Now we can apply the proof of (c) in part (iii) of Lemma 7 to show that the expression in the third line is negative if $v_i''' \leq 0$. But then, (a)-(c) hold, which implies a unique root $\mu^{**} := \arg_{\mu\in(1,\mu^*)}\left\{\Delta_{\widehat{\overline{v}}}(\mu) = 0\right\}.$

(ii) We note that $\widehat{\overline{v}}(\mu^{**}) - \overline{v}(\mu^{**}) = \left[\left(\widehat{W}_N - W_N^*\right)\big|_{\overline{v}=\overline{v}^-(\mu^{**}),\mu=\mu^{**}} + \Delta_F\right]h^{-1} = \Delta_F h^{-1} > 0$, where $\left(\widehat{W}_N - W_N^*\right)\big|_{\overline{v}=\overline{v}^-(\mu^{**}),\mu=\mu^{**}} = 0$ follows from the fact that for $\left(\overline{v}^-(\mu^{**}), \mu^{**}\right)$ we have $\widehat{\psi}_L = 0$ and $\widehat{x}^N = x^N$ with $x \in \{c_L, c_H, a_L, n\}$. At the same time $\widehat{\overline{v}}(1) - \overline{v}(1) = 0$. We can distinguish two cases. Either $\widehat{\overline{v}}(\mu) - \overline{v}(\mu) > 0\ \forall\mu \in (1, \mu^{**})$. In this case, $\mu^{***} = 1$. Alternatively, we can have $\widehat{\overline{v}}(\mu) - \overline{v}(\mu) < 0$ for some $\mu \in (1, \mu^{**})$. But then by continuity there must exist an $\mu^{***} > 1$ such that $\widehat{\overline{v}}(\mu) - \overline{v}(\mu) > 0$ $\forall \mu \in (\mu^{***}, \mu^{**})$. This completes the proof.

Appendix – Figures

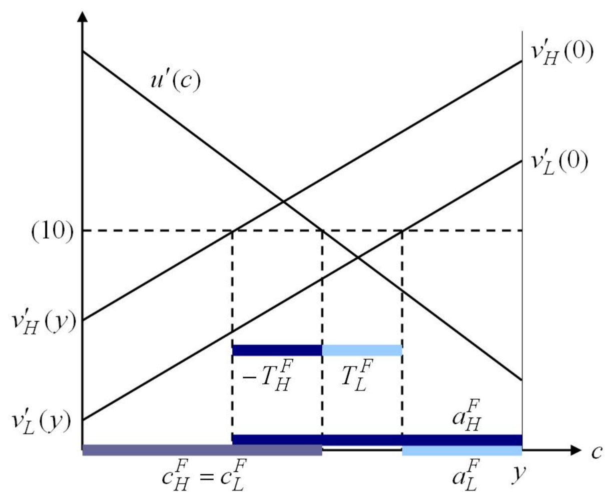

Figure 1: The first-best care and transfer policy with family care only.

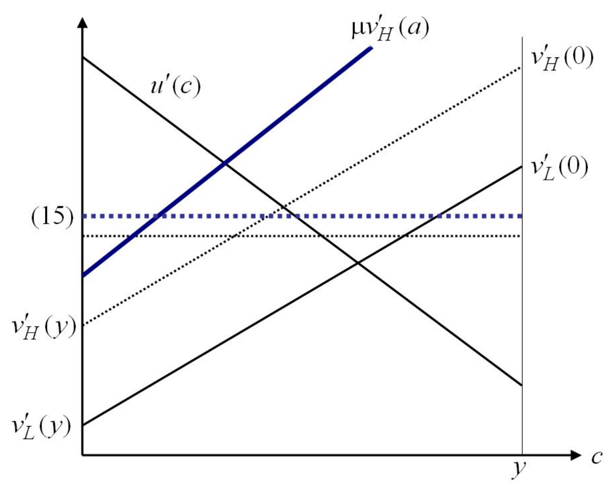

Figure 2: The first-best care and transfer policy with nursing homes.

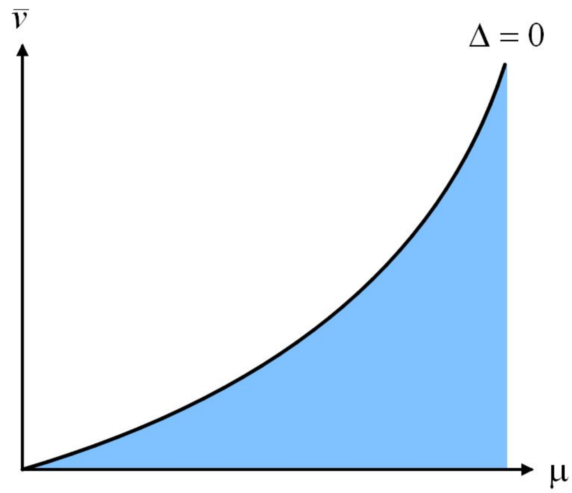

Figure 3: First-best efficient nursing home provision.

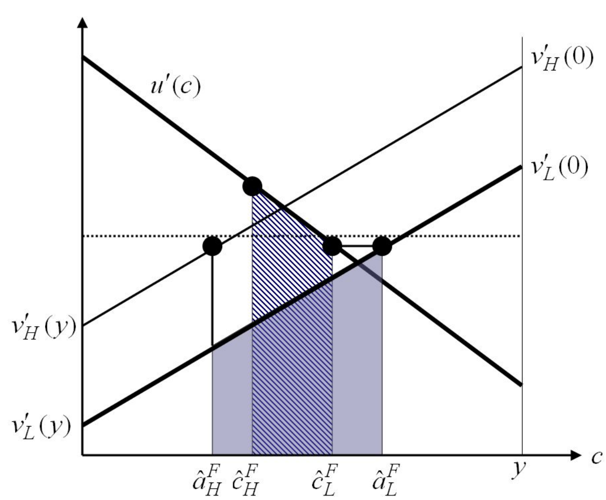

Figure 4: The ‘intuitive’ second-best solution with family care only.

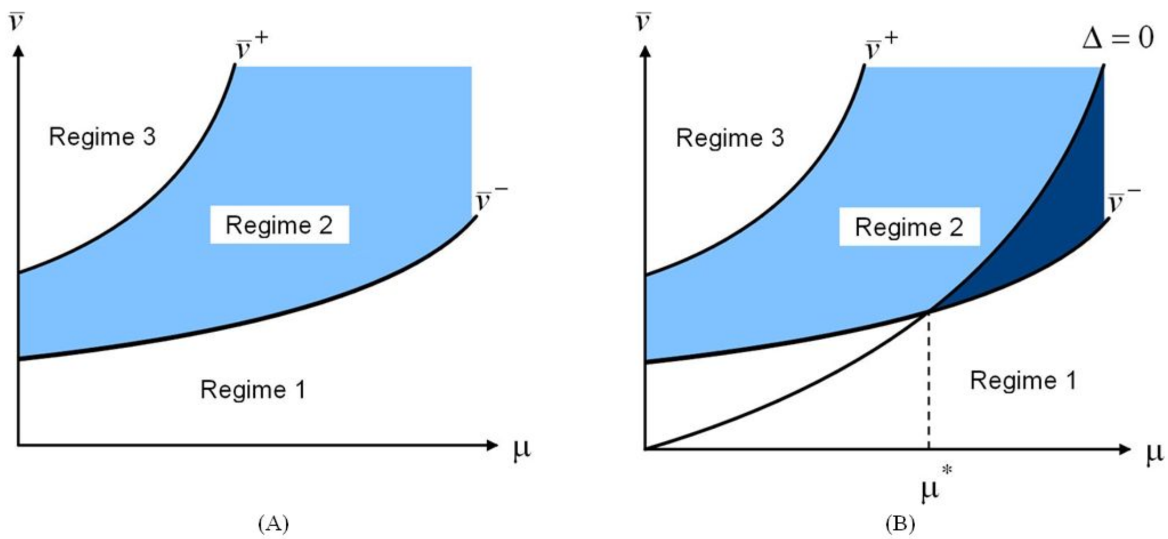

Figure 5: The three regimes and first-best efficient nursing home provision.

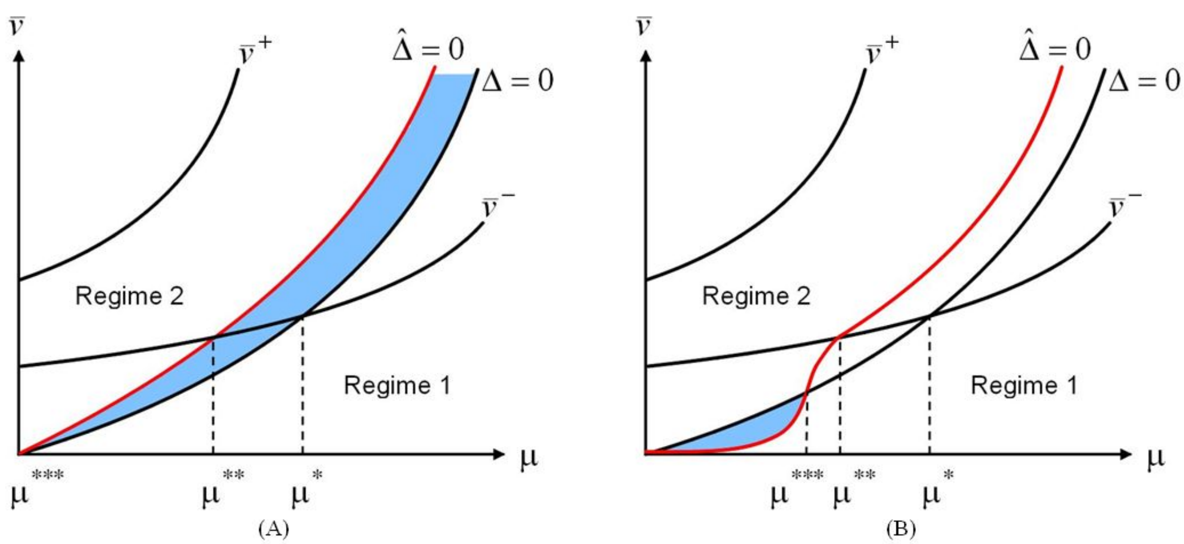

Figure 6: Second-best efficient nursing home provision. Panel A: an over-provision result. Panel B: an under-provision result.